Evaluation of mean waiting time in the system with vacations

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***Abstract*- The paper analyses the system with vacations with deterministic busy and vacation periods, constant service times and Poisson input stream. For this system, we derive approximate formula describing mean waiting time as a function of mean waiting time of the equivalent system without vacations, lengths of busy/vacation periods and service time. The accuracy of the formula is checked by comparing with the simulation.**

Keywords- system with vacations, mean waiting time

I. INTRODUCTION

In the paper we provide approximate analytical formula for the mean waiting time in the system with vacations. Such system is currently implemented in the System IIP [1]. The System IIP provides virtualized network infrastructure for Future Internet. It able us to set a number of, so called, Parallel Internets (PI), working in isolation and sharing common physical infrastructure. For establishing separate virtual links delegated to particular Parallel Internets, we manage access to a physical link by a cycle-based scheduler, as depicted on Fig.1.

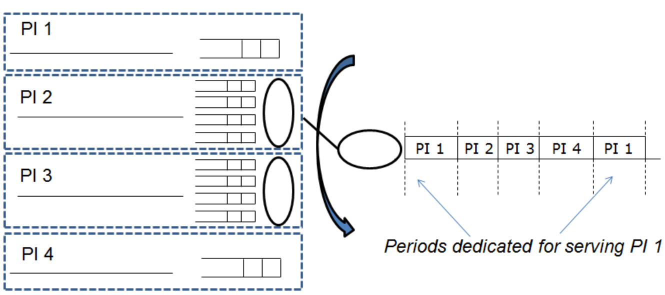

Fig. 1 Cycle-based scheduler for creating virtual links

According to the best knowledge of the authors, such system was not analyzed in the literature. The most of the papers, as e.g. [2], [3], [4], deal with TDMA systems, in which data are transmitted only in the chosen time-slots.

II. ANALYSIS

A. *The system*

The considered queuing system is depicted on Fig. 2. This system belongs to the family of the systems with vacations. It means, that periodically the system is in the busy and the vacation periods. During the busy periods (T_B) the tasks are served while during the vacation periods (T_V) the service is not available. Moreover, we assume infinite buffer size in the system. The queuing discipline is assumed to be FIFO.

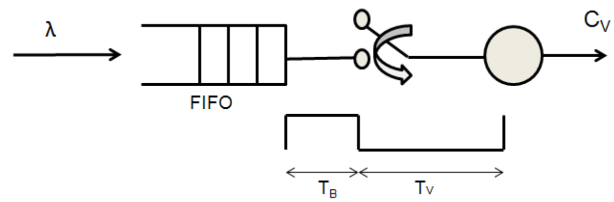

Fig. 2 The system with vacations

Additional assumptions of the system are the following:

- The tasks arrive to the system accordingly to the Poisson process with the rate λ;
- The busy (T_B) and the vacation (T_V) periods are constant;
- The link capacity is equal to C_V bps;
- Service time (h) of the tasks is constant;
- $T_B = n \cdot h$ (n=1, 2, …).

So, in the system the mean available link capacity C for serving incoming task is

$$C = C_v \cdot \frac{T_B}{(T_B+T_v)}. \quad (1)$$

B. *Analysis*

The objective of our analysis is to derive a formula describing the mean waiting time, denoted as $E[W_V]$. The waiting time is defined as a time between the time of tasks arrival, and the time when transmission of this task starts.

Notice, that when the system is fully available (T_V=0), we have M/D/1 system, where the mean waiting time is calculated for well-known Pollachek-Khinchin formula

$$E[W_F] = \frac{\varrho\, h_{res}}{1-\varrho}, \quad (2)$$

where $\varrho = \lambda h$ and h_{res} is the residual service time (in the case of h = *constant*, $h_{res} = \frac{h}{2}$).

So, we can expect that in the system with vacations (with link capacity C_v) is greater than in the equivalent system without vacations (with link capacity C calculated from (1)).

Let we start our analysis from the point of view of the test task arriving to the system. Thanks to the PASTA principle, this test task sees the system at a random moment. This task can arrive when the system is on the busy period or on the vacation period. When the task arrives during the vacation period it should wait for transmission at least (if no other tasks in the system) by the time being the remaining time of the period T_V. On the other hand, when the task arrives during the busy period it can be served immediately (when no others tasks in the system) only in the period (T_B – h). More precisely, when the task joins the system in the last part of the busy period that is smaller than h, it should wait for its transmission even when no other tasks. Let us define:

$$P_V = \frac{T_V}{T_B+T_V}, P_B{}' = \frac{T_B-h}{T_B+T_V}, P_h = \frac{h}{T_B+T_V}, \quad (3)$$

where P_V, P_B', P_h denotes the probability that a task arrives during the vacation period, the busy period (without the last part equal h), and the last part (equal h) of the busy period, respectively.

Our approximate formula for the mean waiting time has the following form:

$$E[W_V] = P_B{}' \cdot E[W_F] + P_V \cdot (T_{Vres} + E[W_F]) + P_h \cdot (h_{res} + T_V + E[W_F]), \quad (4)$$

where $E[W_F]$ is the mean waiting time for the equivalent system and is done by (2) and $T_{Vres} = \frac{T_V}{2}$.
The formula (4) can be simplified to the following form:

$$E[W_V] = E[W_F] + \frac{(T_V+h)^2}{2(T_B+T_V)}. \quad (5)$$

Unfortunately, the formula (4) is not proved in a pure mathematical way but it was only deduced. We assumed that if the task arrives during the busy period it expects similar delay as in the equivalent system without vacations. Complementary, when the task arrives at the periods when it cannot be transmitted immediately (when no other tasks) it should wait for the moment when the busy period starts. In this formula we do not take in a direct way the situation e.g. when a task should wait a number of the busy periods until it starts transmission.
Anyway, the formula (5) is relatively simple and it takes into account in a direct way the impact of the length of the busy and the vacation periods on the task delay comparing to the equivalent system without vacations.

C. Comparing with simulation

In this section we show the accuracy of the formula (5). For this purpose, we do a comparison between the analytical (obtained by the formula (5)) and the simulation results for some exemplary cycle durations. The numerical results were obtained for h=1.
In Table I we present the values of the mean waiting times for two cases when the cycle is rather short, for T_B/T_V=2h/4h and T_B/T_V=10h/20h. The results are obtained for different values of the of traffic load ϱ. One can observe that for these cases, the analytical results are very close to the simulation results and the difference is only a few percentage.

TABLE I
COMPARISION OF MEAN WAITING TIME (SHORT CYCLE)

	TB/TV=2h/4h			TB/TV=10h/20h		
ρ	anal	sim	diff	anal	sim	diff
0,2	2,5	2,4	3%	7,7	7,9	-2%
0,4	3,1	3,0	4%	8,4	8,6	-3%
0,6	4,3	4,2	3%	9,6	9,8	-2%
0,8	8,1	7,9	3%	13,4	13,3	0%
0,9	15,6	15,4	1%	20,9	20,6	1%
0,94	25,6	25,2	2%	30,9	30,6	1%
0,96	38,1	37,3	2%	43,4	43,5	0%

Similarly, the Table II presents the values of the mean waiting times in the case when long cycles are assumed, for T_B/T_V=50h/100h and T_B/T_V=100h/200h. For these cases, we observe worst accuracy of the analytical formula comparing to the previously reported results but it is still on the acceptable level. The difference is about 15% for the most of the studied cases. Notice, that in practical systems, when we apply the cycle solution in the access to the shared link, we rather will design short cycles than long ones since we will try to obtain possibly low delay.

TABLE II
COMPARISION OF MEAN WAITING TIME (LONG CYCLE)

	TB/TV=50h/100h			TB/TV=100h/200h		
ρ	anal	sim	diff	anal	sim	diff
0,2	34,4	36,5	-6%	67,7	72,2	-6%
0,4	35,0	39,4	-11%	68,3	77,8	-12%
0,6	36,3	42,7	-15%	69,6	84,4	-18%
0,8	40,0	47,5	-16%	73,3	92,3	-21%
0,9	47,5	54,6	-13%	80,8	100,1	-19%
0,94	57,5	63,9	-10%	90,8	110,0	-17%
0,96	70,0	76,1	-8%	103,3	122,2	-15%

The accuracy of the formula (5) was also verified for other values of cycle durations and the results were similar to the above presented.

III. SUMMARY

In the paper we have presented the approximate formula for mean waiting time in the system with vacations. The accuracy of the analytical solution is on the satisfactory level, especially when the cycle durations are low, less than 30h.
The formula (5) is very useful to dimension the System IIP.

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