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Planarity of Data Networks

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www.topology-zoo.org

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Introduction

- Network graphs determine many of the properties of a communication network:
 - Reliability
 - Performance
- Developing new network algorithms effectively requires either:
 - Access to a large network
 - Access to an accurate simulation of a large network
- The models that we use should explain how networks are designed and, ultimately, what we see.
- Are there universal laws for network formation that we can use to make design, engineering and maintenance easier?

Modeling Networks

- Which models produce the best approximations of real networks?
- There are many to choose from:
 - Erdos-Renyi and Gilbert
 - Waxman
 - Power-law graphs
 - Highly Optimized Tolerance
- We need a lot of data to identify and classify features that we can use as measures of accurate approximation.

Internet Topology Zoo

- The Internet Topology Zoo (www.topology-zoo.org) is an ongoing research effort to *collect, classify* and *distinguish* “real” networks based on their published images.
- How accurate is this?
 - Based on network ground truth
 - Manual production limits final accuracy
 - But these are **public** documents
 - Worse case is that they are partially idealised.

Classification

- Once we have added networks to the Zoo, we can also capture all of the associated metadata.
 - Are these Commercial or Research and Education networks?
 - Which countries do they occupy?
 - How large is their geographical span?
 - Where are their nodes?
 - Which roles do they support?
 - Which networking layer are they providing?
- The majority of networks document Points-of-Presence (PoPs), rather than pure router-level.
 - These networks may appear less complex than a router-level network.

Graph Analysis

- As we have constructed a large set of graphs, we can now analyse these mathematically, to identify matches between similar graph attributes, or features, and metadata and classifications derived from the original maps.
- Such measures include node degree, assortativity and planarity – the focus of this presentation.
- A *planar graph* is one that can be drawn in a plane without edges crossing.
- We are less interested in what it means to be planar, in this context, rather we want to establish what it can tell us about synthesizing simulated network graphs on the large scale.

Planarity Analysis of the Zoo

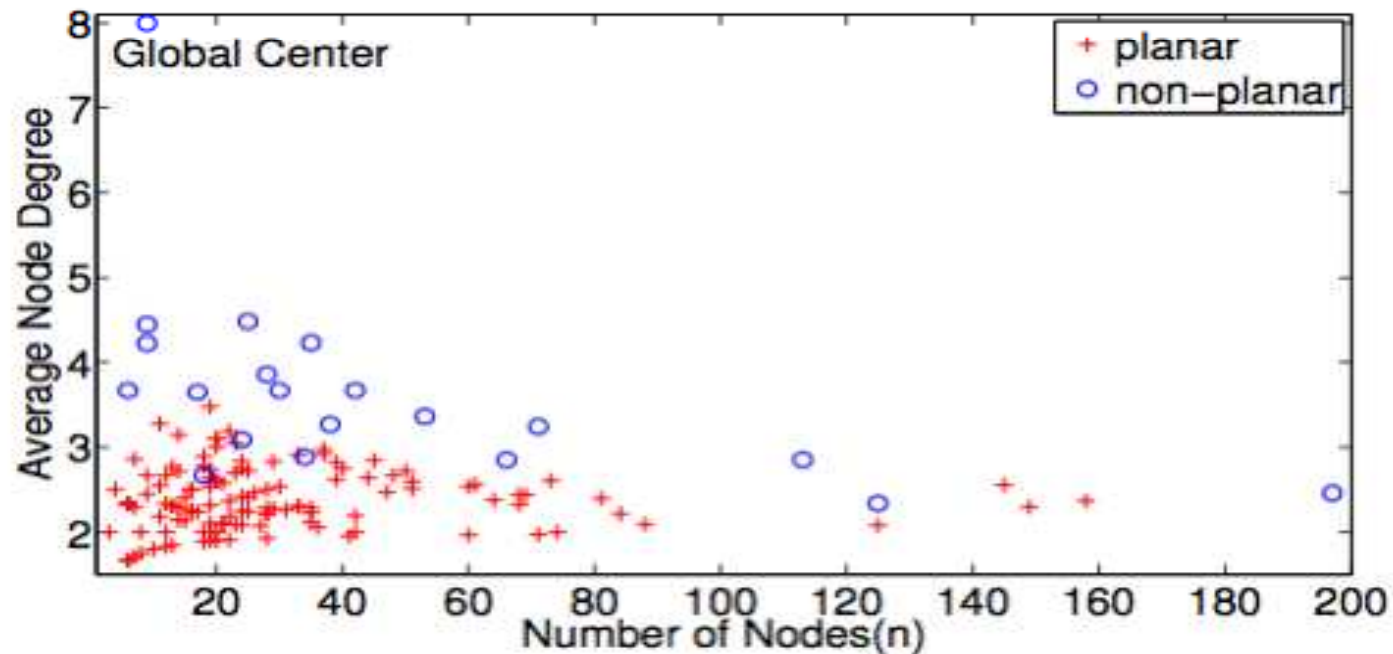
- 147 transcribed networks were used, from Zoo.v0.01.
- If a graph had disconnected elements, we analyzed the largest connected components.
- Multi-edges were converted to single-edge.
- Planarity analysis was carried out using the Biyer-Myrvold planarity test algorithm. ($O(n)$)
- Of the 147 networks, 21 (14%) were non-planar.
- The obvious question is “What does this mean?”

Noteworthy first results

- Layer 1 networks are slightly more likely to be planar.
- All of the Research and Education Networks are planar.
- Networks who provider customer-oriented services, with colocation facilities or per-customer state, are more likely to be non-planar.
- Larger networks, in terms of geographic span, are more likely to be non-planar.

Expectations: Impact of Size

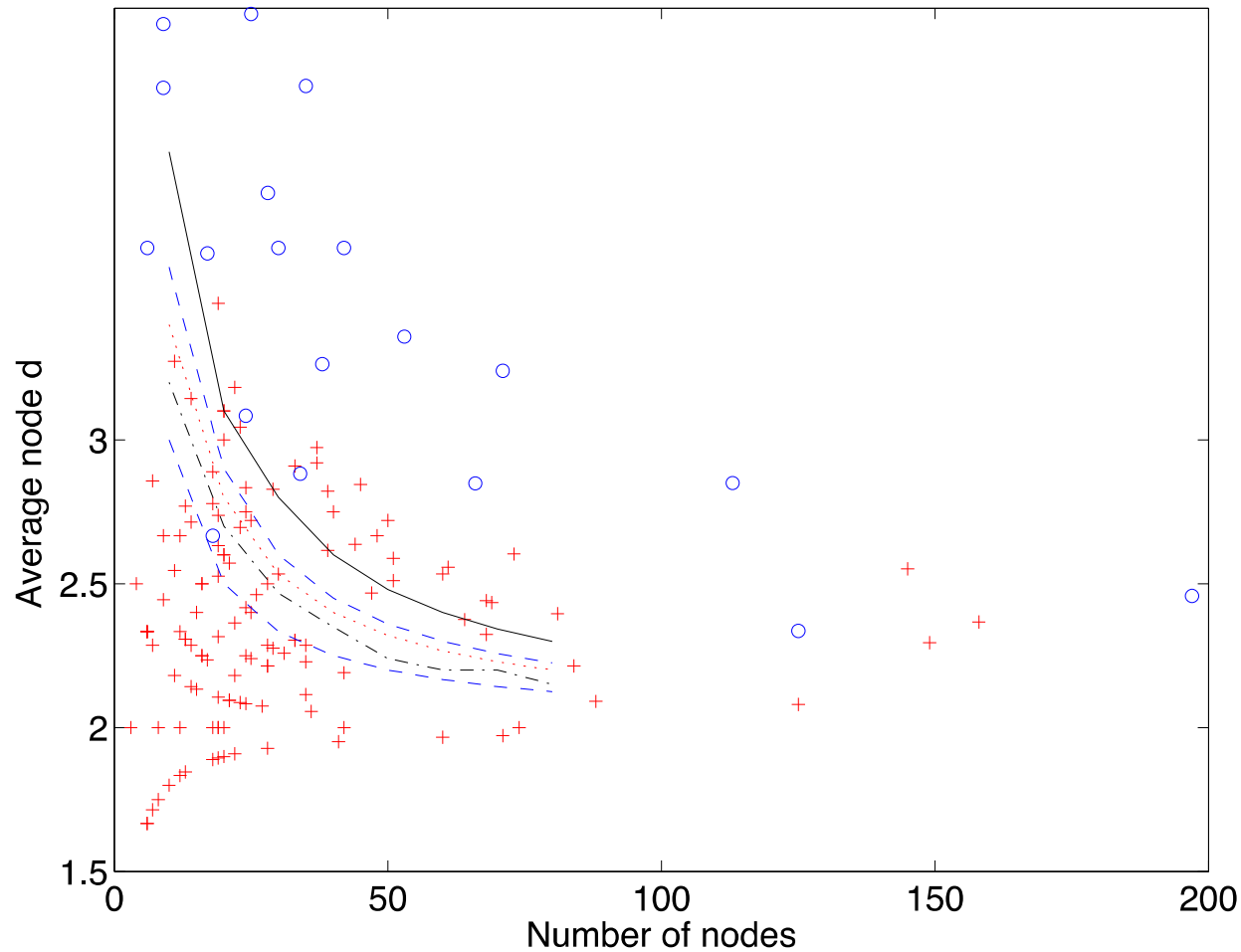
- We would expect larger, more complicated, networks to be more likely to be non-planar.
- Therefore investigate Average Node Degree versus Number of Nodes:



Are these results expected?

- We need to compare these results to other, large, bodies of graph information in order to see if our results are to be expected or unusual.
- Two approaches:
 - Generate a large body of truly random connected graphs.
 - Generate a large body of random, but optimised, graphs that more closely approximate real networks.
- We implement the first approach with Erdos-Reyni random graphs and connecting each node in turn in a way that simulates a given average node degree and network size – then measure their planarity.
- We add contours to the resulting plot to show the 95% confidence intervals for increasingly non-planar behaviour.

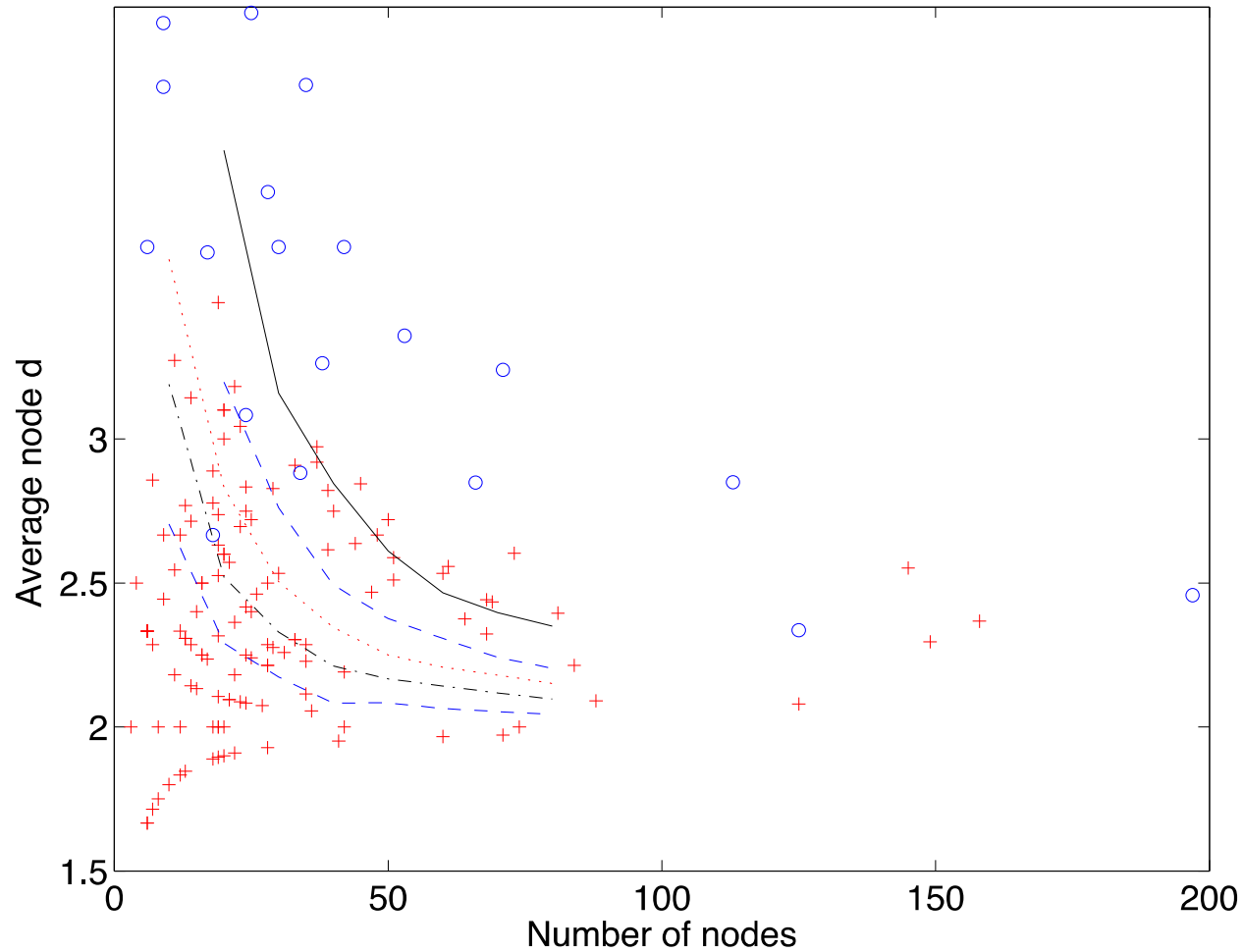
Planarity as a function of average node degree and network size, with contours. (Random Graphs)



Optimised Graphs

- We expect real networks to be optimised with respect to cost and service provision.
 - Real networks aren't random connections!
- The mechanism that we used is in the paper, but we can see a change in the graphs.

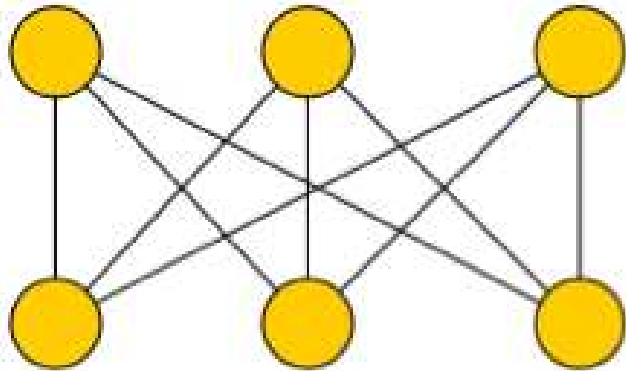
Planarity as a function of average node degree and network size, with contours. (Optimised Graphs)



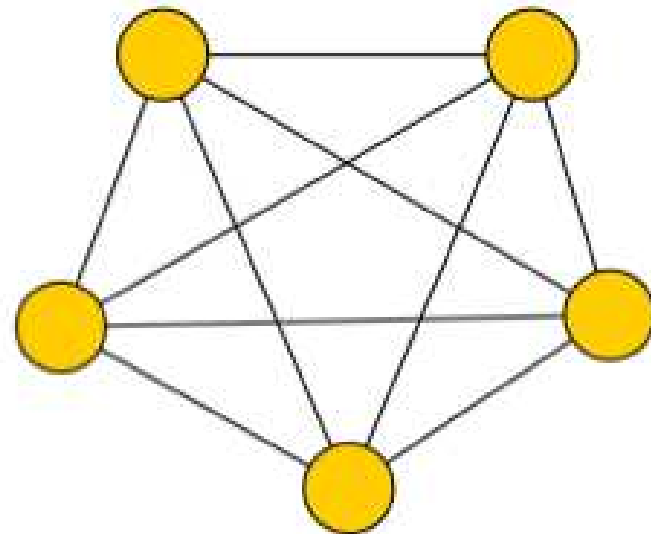
Analysis

- We appear to see almost no non-planar graphs until the point at which, from random or random+optimisation graphs, we should have already been seeing a majority of non-planar graphs.
- Neither solution is a good match to what we see in the Zoo.
- Can we compare the types of non-planarity?
 - Restricted to a small set of nodes and predominantly of one type.

K_5 and $K_{3,3}$



$K_{3,3}$



K_5

What is causing this lower rate of non-planarity?

- It appears that, unexpectedly, Internet networks have very high likelihood of being planar.
- Even where non-planarity occurs, it is in the vast minority of the nodes.
- Layer 1 networks are, we know, more likely to be planar.
 - Is our planarity a L2,3+ tracing effect?
- Is it because planar graphs are easier to draw?
- Is design carried out by eye?
- Is this evidence of a real lack of automated optimisation?
- Is there selection bias in the Zoo?
- Planarity is more likely to be a signature of our desire for simplicity, than a specified design constraint.

Conclusions

- There is a high degree of planarity in observed networks.
- Do we need to consider more complex optimisation objectives? If so, how and to what aim?
- We seek to understand why current networks look as they do, in order to better understand design of real networks and accurate simulation.
- Optimisation should still be pursued, but we need to determine the hidden objectives that appear to be leading to a very distinct construction style in contemporary networks.

Questions

- Thank you!