A Game Theoretic Analysis of Selfish Content Replication on Graphs

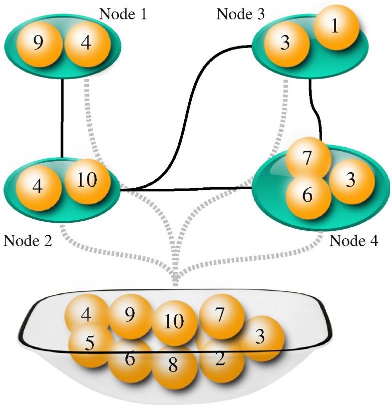
Valentino Pacifici, György Dán

Laboratory for Communication Networks
School of Electrical Engineering
KTH, Royal Institute of Technology
Stockholm - Sweden

San Francisco, September 7, 2011

The problem of content replication

Scenario



Questions:

- ∃ satisfying allocation for every node?
- will the nodes be able to reach it?

- No central authority ⇒ Selfish nodes

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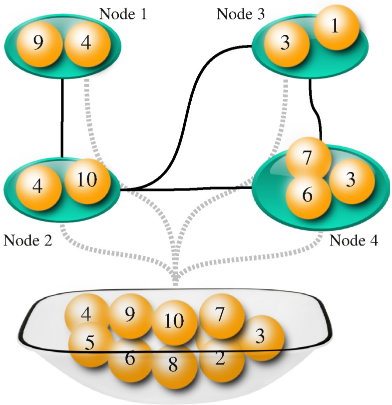

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- Network caches
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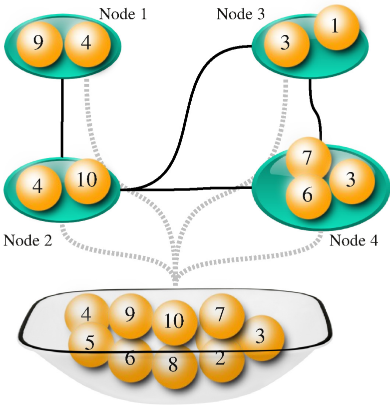

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The Model

- Replication game: $< N, (\mathcal{R}_i), (U_i) >$
  - N is the set of players
  - $\mathcal{R}_i$ is the action set of player i
  - $r_i \in \mathcal{R}_i$, $r_i = (r_i^1, \ldots, r_i^{|\mathcal{O}|})$ is an action of player i
  - U_i is the utility function of player i
- A *strategy* is the choice made by player $i \in N$
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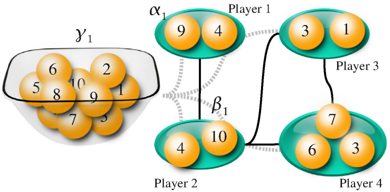

- $\alpha_i \leq \beta_i < \gamma_i$
- $w_i^o \in \mathbb{R}_+$ is the demand for object $o \in \mathcal{O}$ at node $i \in N$
- $U_i^o(1, r_{-i}) = \begin{cases} w_i^o [\gamma_i - \alpha_i] & \text{if } \pi_i^o = 1 \\ w_i^o [\beta_i - \alpha_i] & \text{if } \pi_i^o = 0 \end{cases}$

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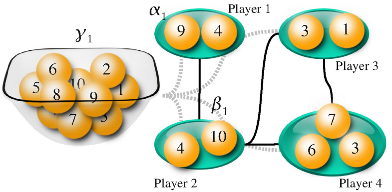

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- **Nash Equilibrium**: a strategy profile r^* from which no player i wants to deviate unilaterally (i.e. given that the rest of the players play r^*_{-i})

- **Best reply** of player i: the best strategy r_i^* of player i given the other players' strategies

$$U_i(r_i^*, r_{-i}) \geq U_i(r_i, r_{-i}) \;\; \forall \; r_i \in \mathcal{R}_i.$$

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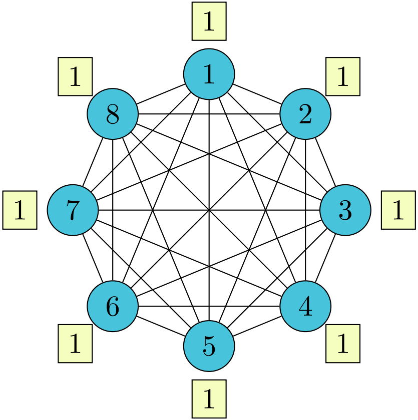

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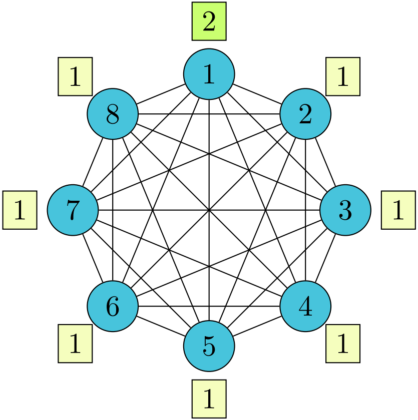

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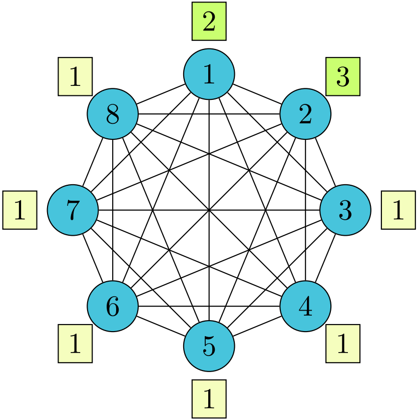

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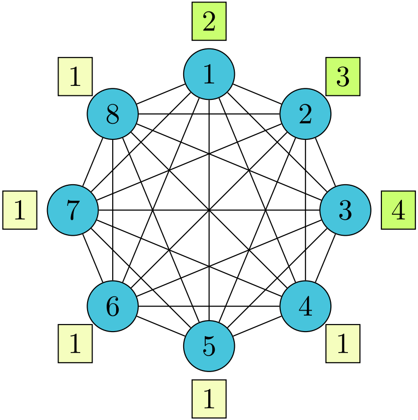

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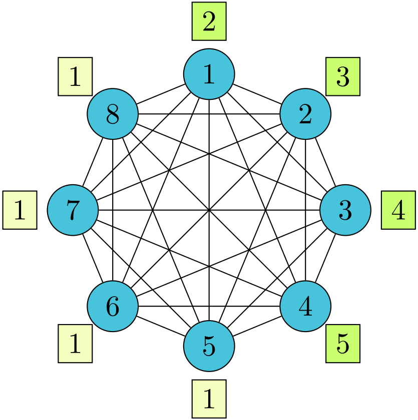

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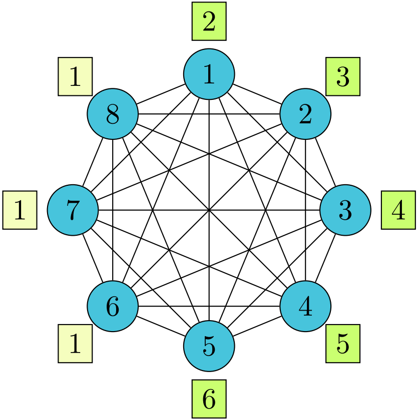

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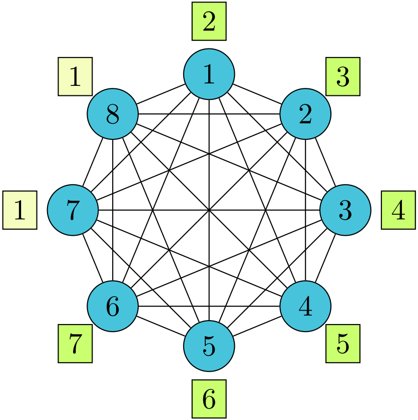

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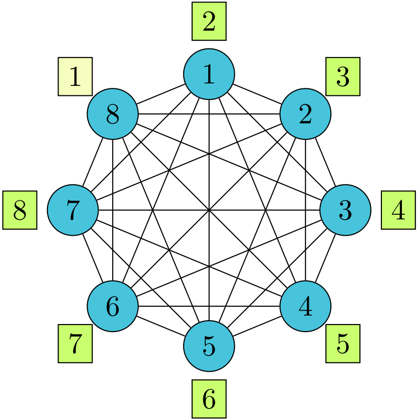

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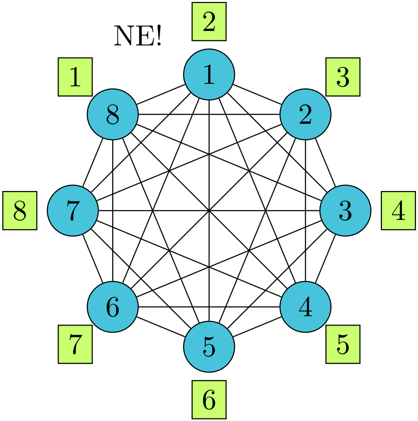

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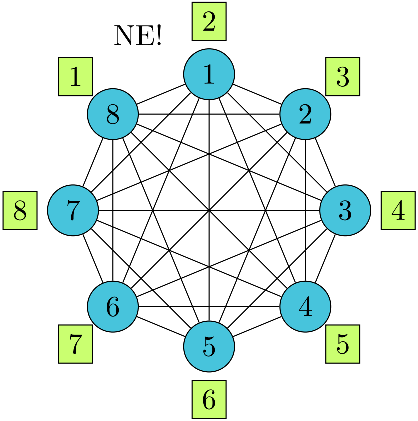

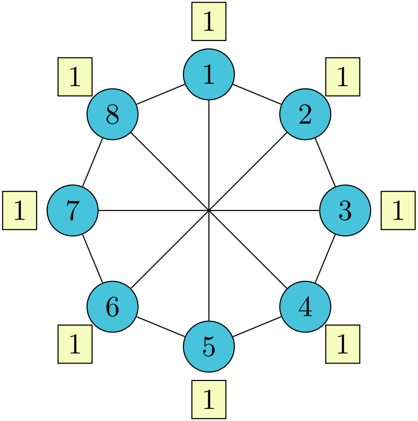

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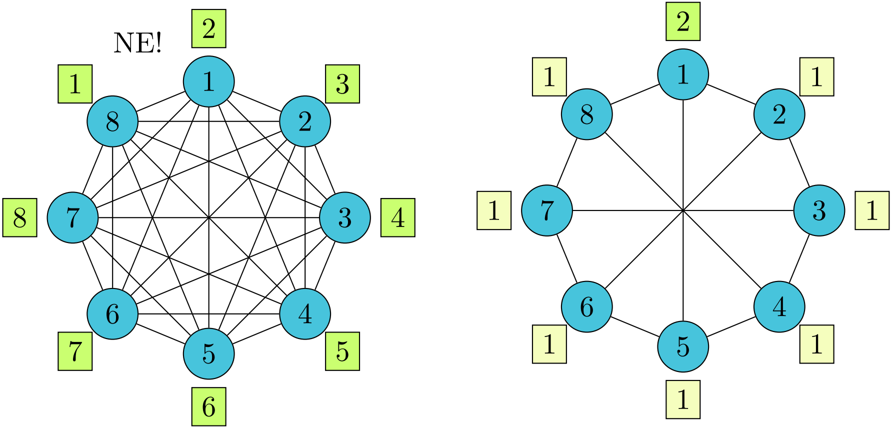

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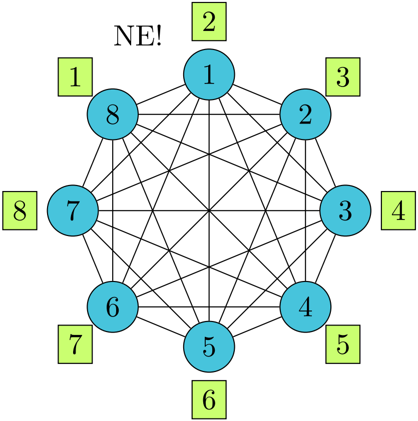

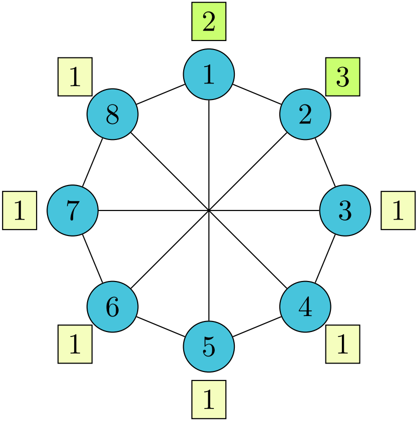

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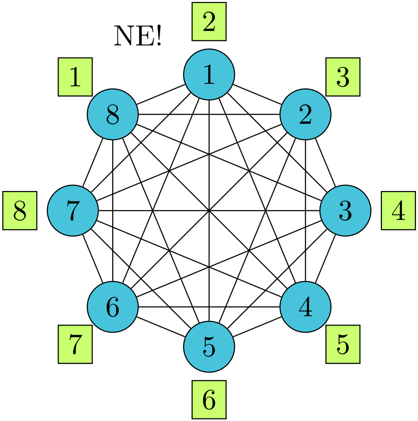

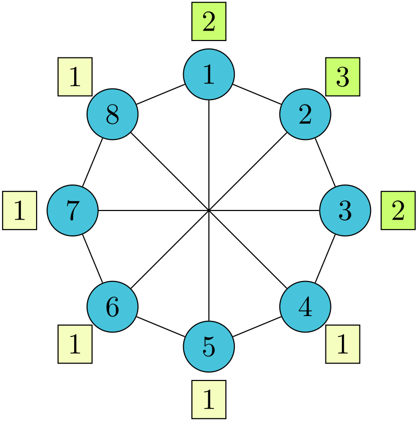

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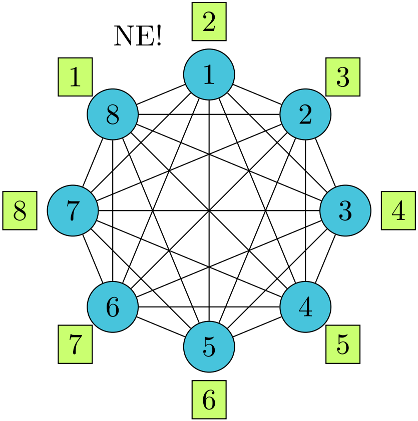

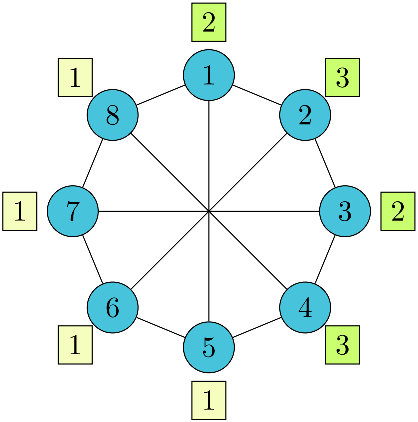

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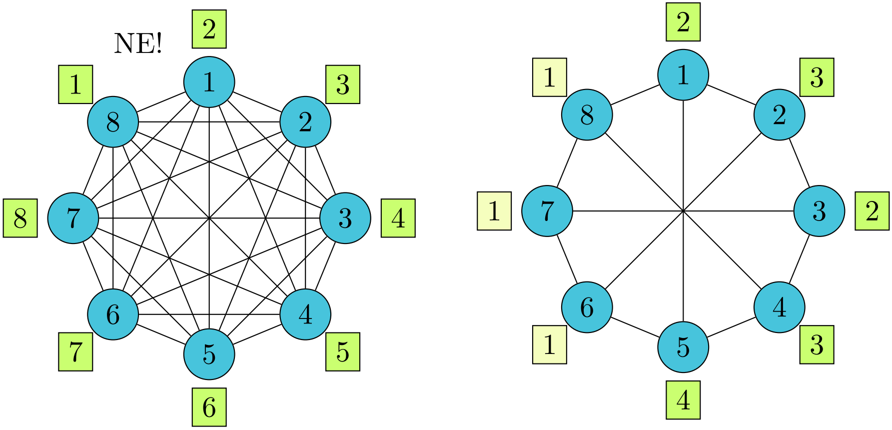

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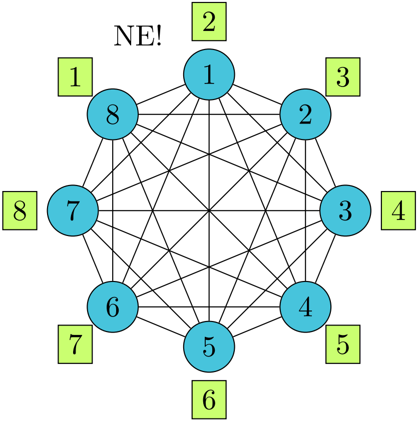

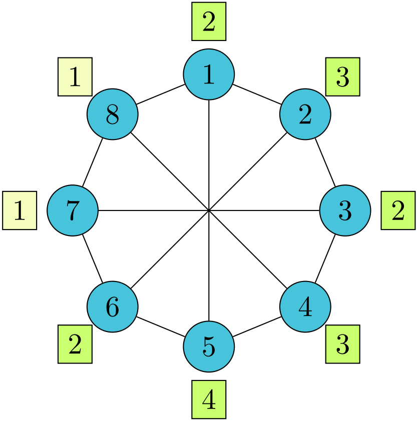

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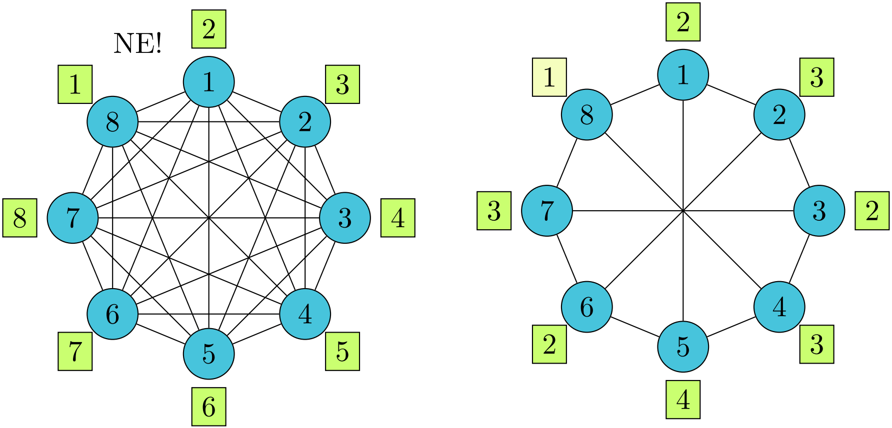

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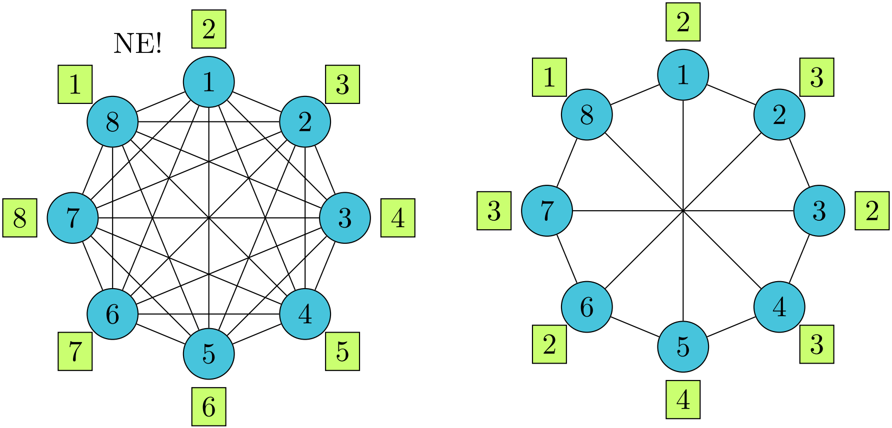

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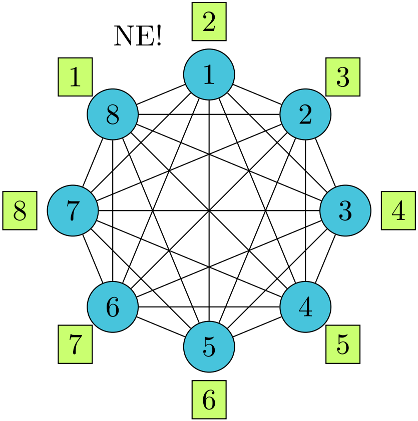

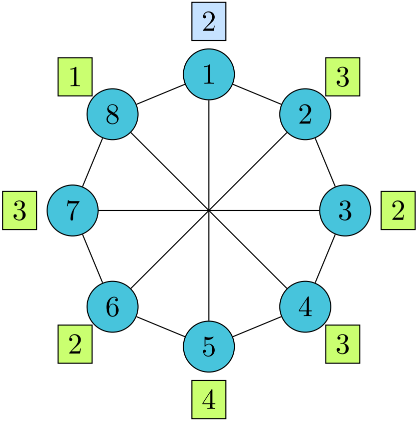

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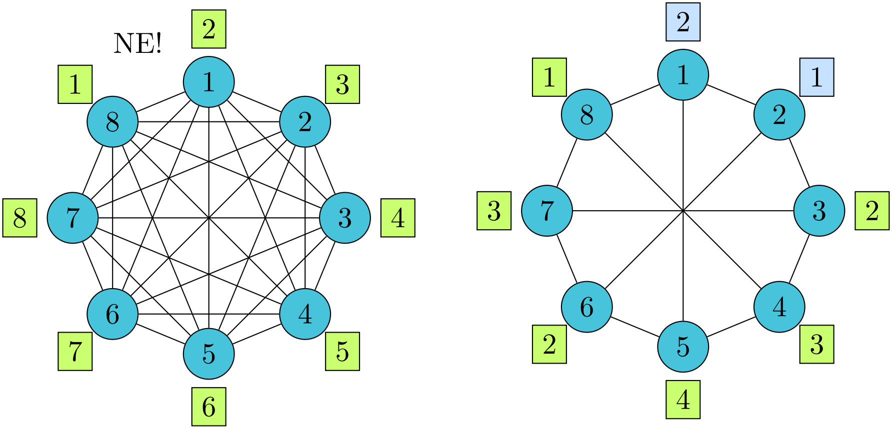

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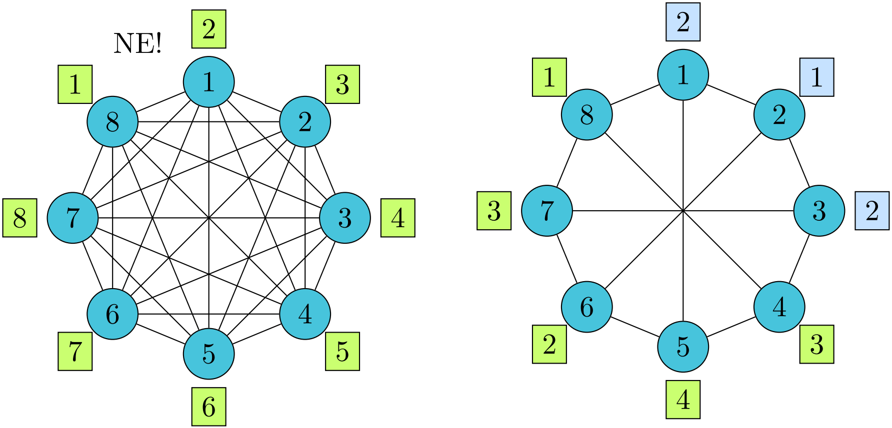

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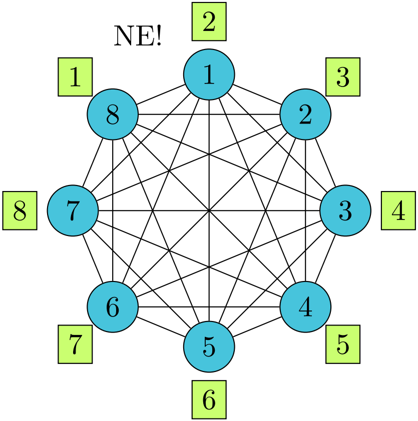

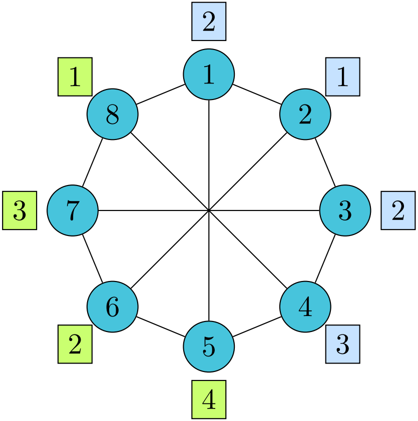

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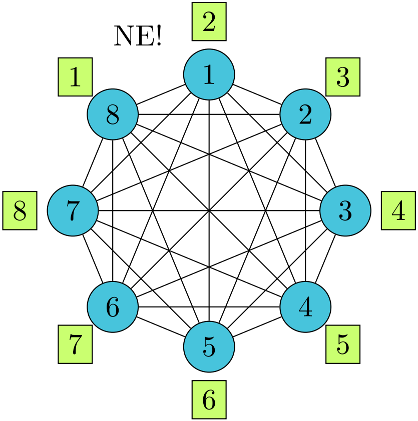

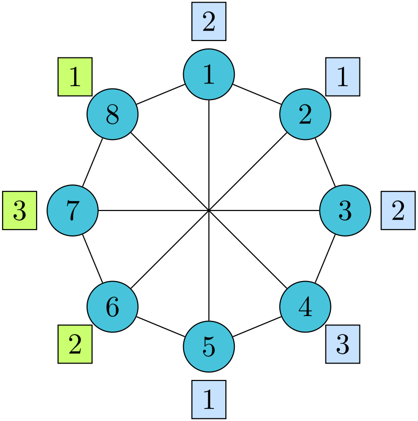

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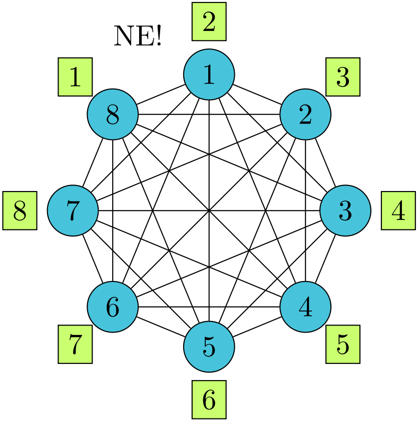

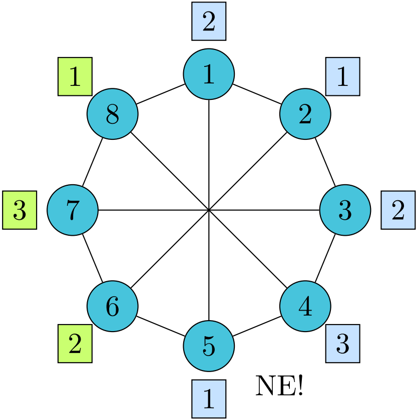

Equilibrium Existence

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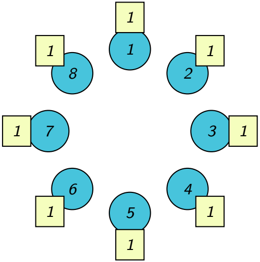

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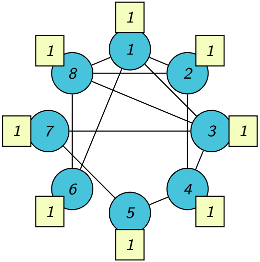

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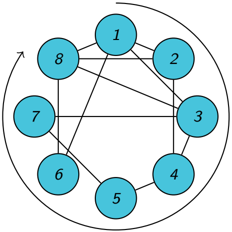

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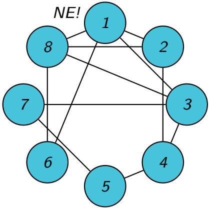

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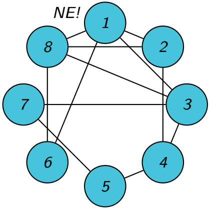

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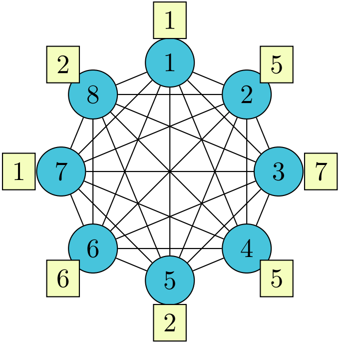

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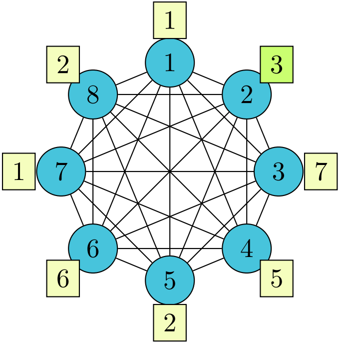

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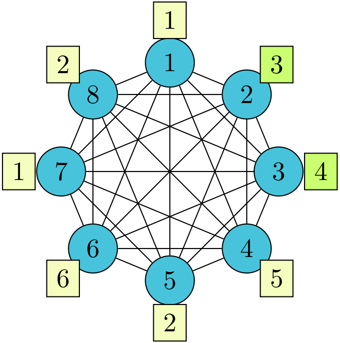

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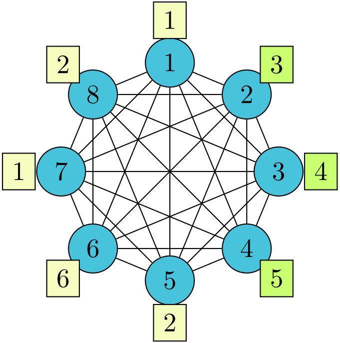

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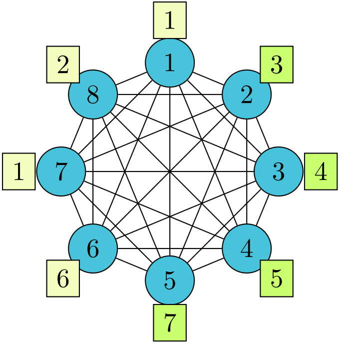

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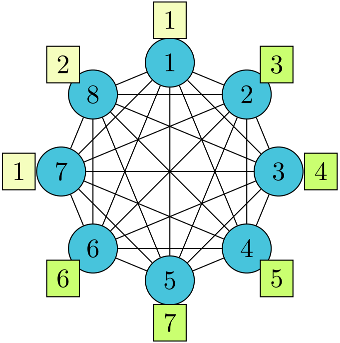

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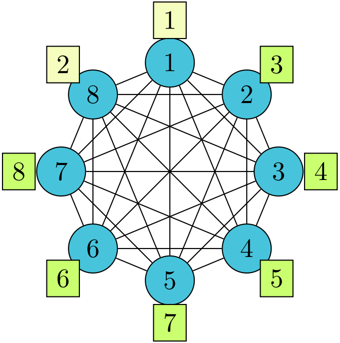

Convergence to a Nash Equilibrium

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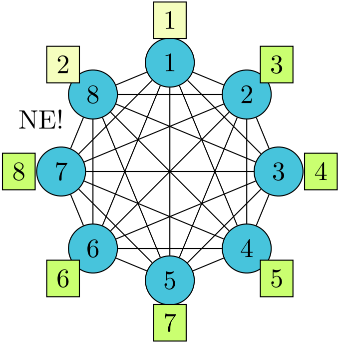

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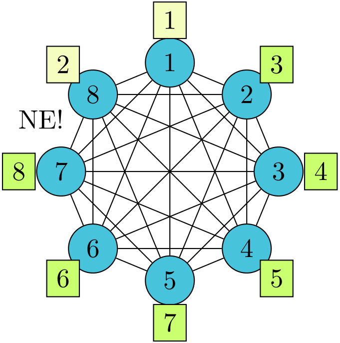
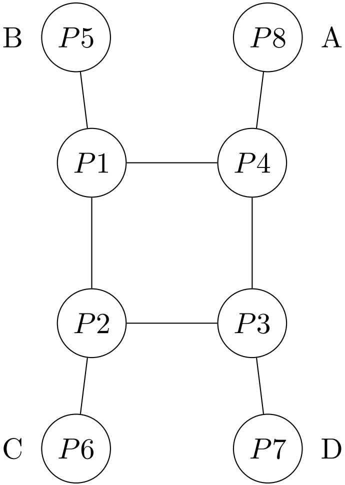


Theorem

Every best reply path in a replication game played over a complete social graph is finite (i.e. does not contain any cycle).

An example of a cycle



Dem.	A<B	B<C	C<D	D<A
Player	*P1*	*P2*	*P3*	*P4*
r(0)	A	B	D	A
r(1)	A	B	C	A
r(2)	B	B	C	A
r(3)	B	B	C	D
r(4)	B	C	C	D
r(5)	A	C	C	D
r(6)	A	C	D	D
r(7)	A	B	D	D
r(8)	A	B	D	A

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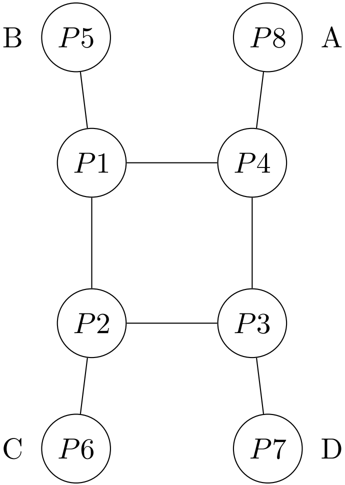

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Theorem

If $K_i = 1\ \forall i \in N$, *a best reply path that leads to a NE always exists.*

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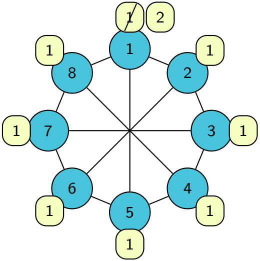

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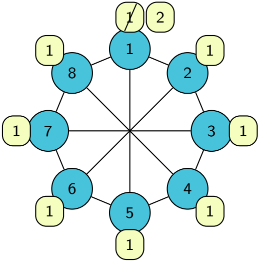
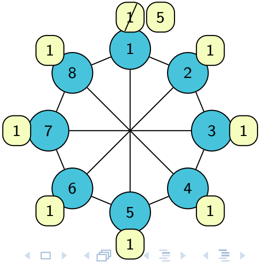


A *lazy improvement step*:



The plesiochronous dynamic

Theorem

If $\beta_i = \alpha_i \; \forall i \in N$ *and player* i *makes an improvement step at time* t *only if no neighboring player* $j \in \mathcal{N}(i)$ *makes an improvement step at time* t*, then every lazy improvement path is finite.*

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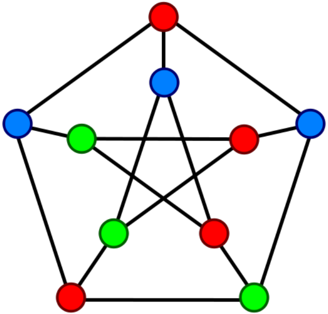

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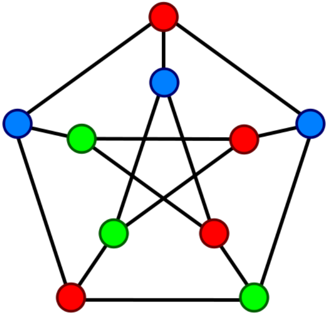

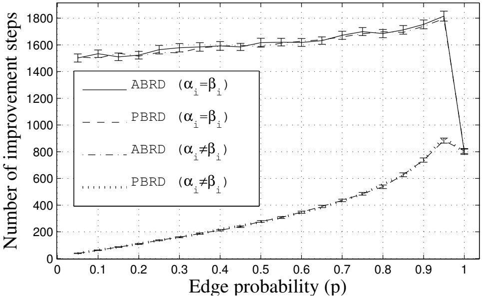

Conclusion and future work

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  - Sufficient condition to reach a Nash equilibrium
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- Future work
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A Game Theoretic Analysis of Selfish Content Replication on Graphs

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Laboratory for Communication Networks
School of Electrical Engineering
KTH, Royal Institute of Technology
Stockholm - Sweden

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