Backlog-Based Random Access in Wireless Networks: Fluid Limits and Delay Issues

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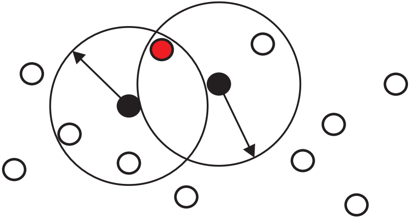
Joint work with Sem Borst, Johan van Leeuwaarden and Alexandre Proutière

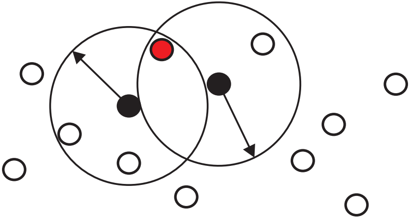
Large-scale wireless networks



- covering large areas, huge numbers of nodes
- centralized control is infeasible
- nodes operate autonomously, and share medium in distributed fashion

Nodes do not just use the network, they are the network

Large-scale wireless networks



Randomized algorithms provide popular mechanism for distributed medium-access control

CSMA (Carrier-Sense Multiple-Access) algorithms

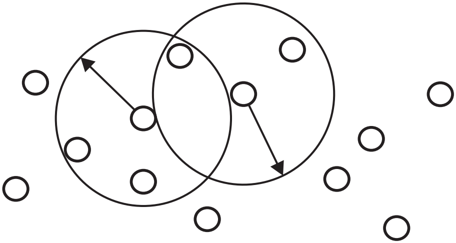
- Each node attempts to activate after random back-off time
- A node activates only if no interfering node is active
- low implementation complexity, but highly complex behavior on macroscopic level

Interference graph

The network is described by an undirected graph

- The vertices represent the various nodes of the network
- The edges indicate which pairs of nodes interfere

[Boorstyn *et al.* (1980), Wang & Kar (2005), Durvy & Thiran (2006)]



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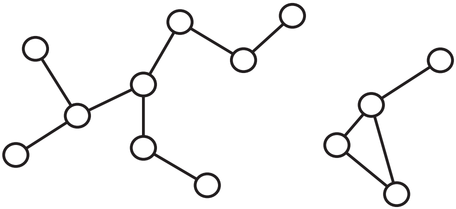

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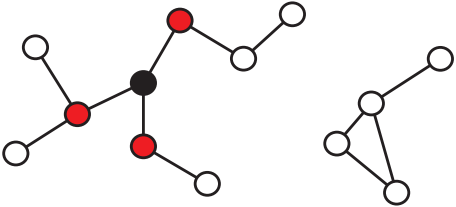


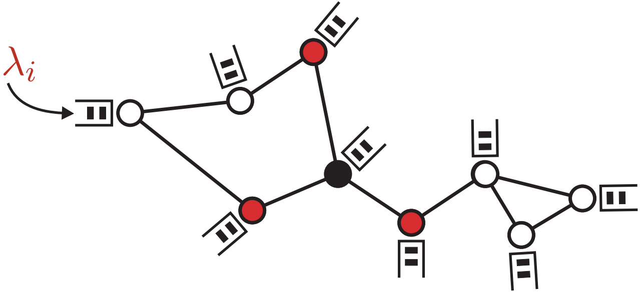
Model description

As opposed to most previous work that assumed saturated buffers, we consider queueing dynamics

Packets arrive at node i according to a Poisson process with rate λ_i

Packet transmission times of node i are exponentially distributed with mean $1/\mu_i$. Once a packet has been transmitted, it leaves the system

Denote by $\rho_i = \lambda_i/\mu_i$ the traffic intensity of node i

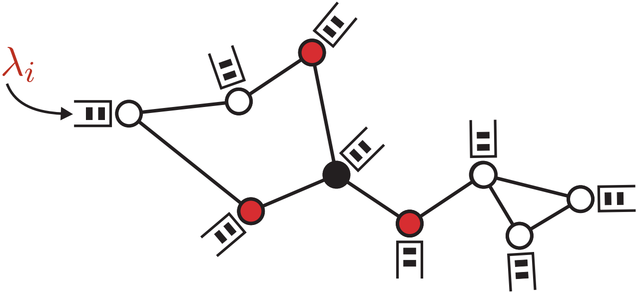


Backlog-based CSMA algorithms

Denote by $L_i(t)$ the number of packets at node i at time t

We consider random CSMA algorithms where

- node i attempts to activate at exponential rate $f_i(L_i(t))$, if inactive. $f_i(\cdot)$ is called the activation function of node i
- node i de-activates at exponential rate $g_i(L_i(t)) = p_i(L_i(t))\mu_i$, if active, with $p_i(L_i(t))$ the probability that node i releases the medium if a packet transmission ends at time t. $g_i(\cdot)$ is called the de-activation function of node i



Markov process

Define $\Omega \subseteq \{0,1\}^M$ as the set of all feasible joint activity states, with M the number of nodes in the network

Let $\sigma(t) = (\sigma_1(t), \ldots, \sigma_M(t)) \in \Omega$ represent the activity state of the network, with $\sigma_i(t)$ indicating whether node i is active at time t or not

Under these CSMA algorithms, $(L(t), \sigma(t), t \geq 0)$ is a continuous-time Markov process, with $L(t) = (L_1(t), \ldots, L_M(t))$

Backlog-based CSMA algorithms

What is the network performance, depending on the functions $f_i(\cdot)$ and $g_i(\cdot)$?

Backlog-based CSMA algorithms

What is the network performance, depending on the functions $f_i(\cdot)$ and $g_i(\cdot)$?

What is the stability region?

What are its delay characteristics?

Stability

$\rho \in \mathrm{conv}(\Omega)$, with $\rho = (\rho_1, \ldots, \rho_M)$, is a necessary condition for stability

For suitable choices of $f_i(\cdot)$ and $g_i(\cdot)$ (e.g., $g_i(n) = 1/(1 + \log(n+1))$ and $f_i(n) = 1$), any network is stable as long as $\rho \in \mathrm{int}(\mathrm{conv}(\Omega))$ [Rajagopalan *et al.* (2009), Jiang *et al.* (2010), Ghaderi & Srikant (2010)]

However, simulation experiments demonstrate that these choices cause excessive backlogs and delays!

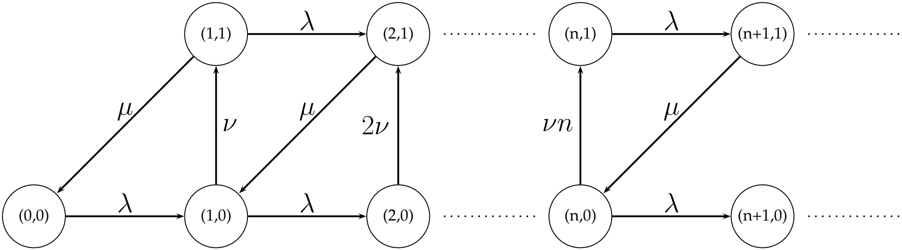
Full interference graph

Assume complete interference graph and $g_i(n) = \mu$ and $f_i(n) = \nu n$

The system can be described by a continuous-time Markov process with state space $\{0, 1, 2, \ldots\} \times \{0, 1\}$

- First component represents the total number of packets in the system, $L(t) = \sum_{i=1}^{M} L_i(t)$
- Second component indicates whether one of the nodes is active

Denote $\lambda = \sum_{i=1}^{M} \lambda_i$ and $\rho = \sum_{i=1}^{M} \rho_i = \lambda/\mu$



Full interference graph

Assume $\rho < 1$ and denote by L the total number of packets in the system in steady state, i.e.

$$\mathbb{P}\{L = n\} = \lim_{t\to\infty} \mathbb{P}\{L(t) = n\}$$

Theorem

The generating function of L is given by

$$\mathbf{E}\{z^L\} = \left(\frac{1-\rho}{1-\rho z}\right)^{\lambda/\nu+1} \mathrm{e}^{(z-1)\lambda/\nu}$$

In particular,

$$\mathbf{E}\{L\} = \frac{\rho + \frac{\lambda}{\nu}}{1-\rho} = \frac{\lambda(\mu+\nu)}{\nu(\mu-\lambda)}$$

General activation functions

Assume $g_i(n) = \mu$

Theorem

For $f_i(\cdot) \equiv f(\cdot)$ a strictly increasing, unbounded and concave function,

$$\mathbf{E}\{L\} \geq \frac{\rho}{1-\rho} + Mf^{-1}\left(\frac{1}{M}\frac{\lambda}{1-\rho}\right)$$

For $f_i(\cdot) \equiv f(\cdot)$ a strictly increasing, continuous and convex function,

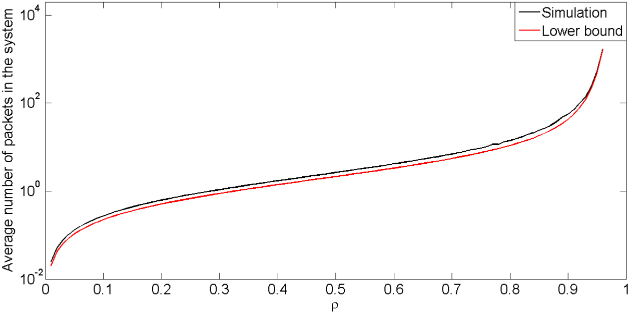
$$\mathbf{E}\{L\} \leq \frac{\rho}{1-\rho} + Mf^{-1}\left(\frac{1}{M}\frac{\lambda}{1-\rho}\right)$$

Note: when $f(n) = \nu n$ the inequalities are in fact equalities

General activation functions

The bounds turn out to be extremely tight

Simulation for the average number of packets in the system for $f(n) = \log(n+1)$ with $M = 4$

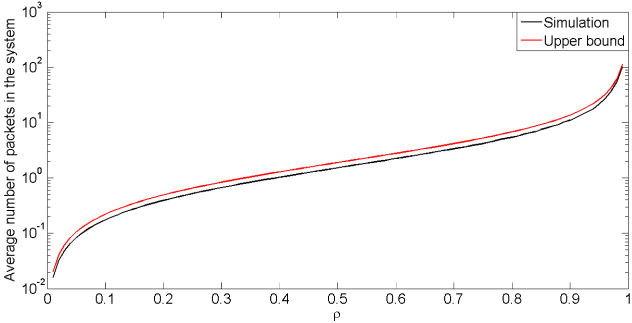


In heavy traffic, as $\rho \uparrow 1$, L grows like $M \exp(\frac{\rho}{M(1-\rho)})$

General activation functions

The bounds turn out to be extremely tight

Simulation for the average number of packets in the system for $f(n) = \mathrm{e}^n - 1$ with $M = 4$



In heavy traffic, as $\rho \uparrow 1$, L grows like $\frac{\rho}{1-\rho}$

General topologies

Assume $g(n) = \mu$ and assume the system to be stable

Using a heuristic argumentation we find

$$\mathbf{E}\{L_i\} \approx f_i^{-1}(\mu\phi_i(\rho)),$$

where $\phi_i(\rho)$ is the constant activity factor needed for a throughput ρ

When $f_i(\cdot)$ increases in a more aggressive manner, the mean number of packets at node i will be smaller

General topologies

Assume $M = 4$ and $\rho_i = \rho/2$

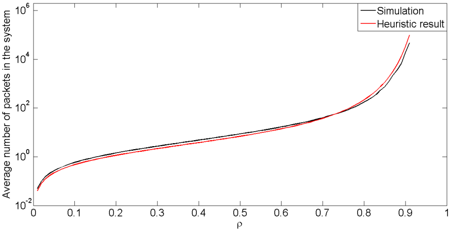
For a ring network we find

$$\mathbf{E}\{L\} \approx \sum_{i=1}^{4} f_i^{-1}\left(\frac{\lambda}{1-\rho}\right)$$

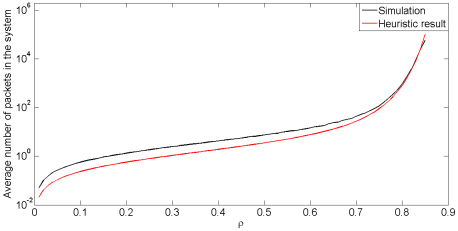
For a linear network we find

$$\mathbf{E}\{L\} \approx f_1^{-1}(\phi_{14}(\rho)) + f_2^{-1}(\phi_{23}(\rho)) + f_3^{-1}(\phi_{23}(\rho)) + f_4^{-1}(\phi_{14}(\rho)),$$

with $\phi_{14}(\rho) = \frac{\lambda}{2(1-\rho)}$ and $\phi_{23}(\rho) = \frac{(2-\rho)\lambda}{4(1-\rho)^2}$



Simulation for $f_i(n) = \log(n+1)$



Fluid limits

More aggressive activation functions can improve the delay performance

Existing stability results involve slowly varying activation functions, e.g. $f(n) = \log(n + 1)$

Depending on the topology, how aggressive are the activation functions allowed to grow?

Examine the dynamics of the Markov process $Z(t) = (L(t), \sigma(t))$ using fluid limits

A fluid limit may be interpreted as a first-order approximation of the Markov process

Fluid limits

Consider a sequence of processes $Z^N(t)$, where the initial states satisfy $\sum_{i=1}^{M} L_i(0) = N$ and $L_i^N(0)/N \to q_i \geq 0$ as $N \to \infty$

The process $\bar{Z}^N(t) = (\frac{1}{N} L^N(Nt), \sigma^N(Nt))$ is the fluid-scaled version of the process $Z^N(t)$

$\xi(t) = \lim_{N\to\infty} \frac{1}{N} L^N(Nt)$ is called a fluid limit

We encounter different types of fluid limits depending on the mixing properties of the activity process $\sigma^N(t)$

These properties depend on the topology and activation functions

Trichotomy

Fast mixing - Deterministic fluid limits

- Transitions between the various activity states are not observed at fluid level
- Slowly varying activation functions

Slow mixing - Inhomogeneous Poisson fluid limits

- Transition times between the various activity states are driven by time-inhomogeneous Poisson processes
- Intermediate activation functions

Torpid mixing - Pseudo-deterministic fluid limits

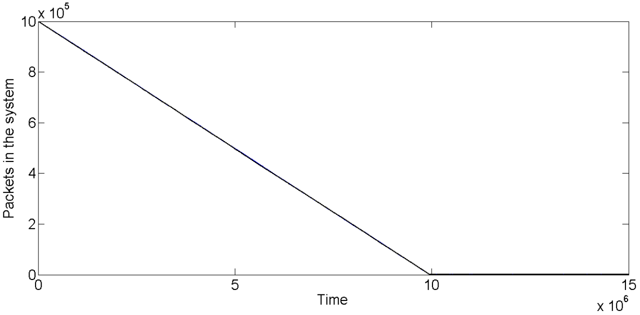
- Activity state seems to be frozen at fluid level
- Transitions only occur when a queue empties
- Aggressive activation functions

Example of fast mixing

2-partite complete interference graph

Subsets have cardinality 1

$f(n) = n$ and $g(n) = 1$

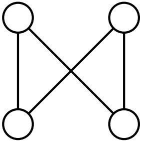


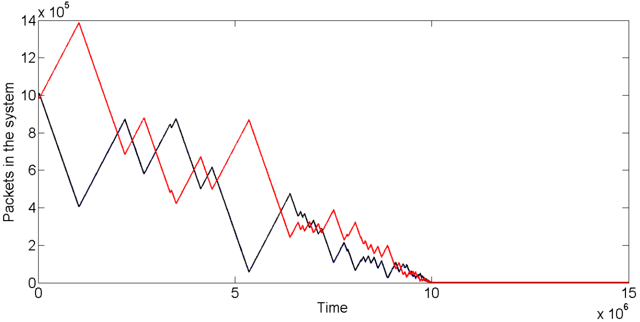
Example of slow mixing

2-partite complete interference graph

Subsets have cardinality 2

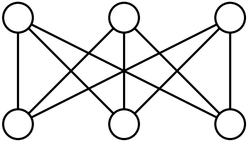
$f(n) = n$ and $g(n) = 1$





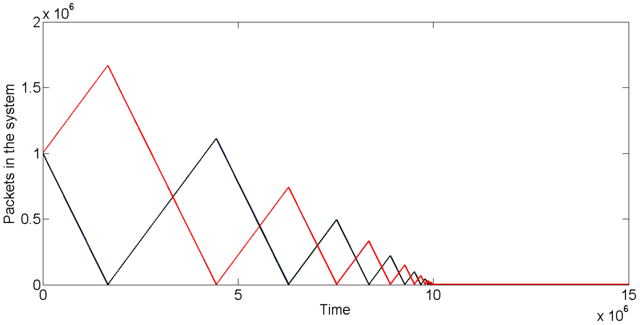
Example of torpid mixing

2-partite complete interference graph



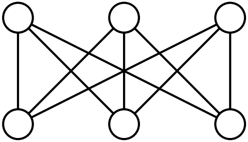
Subsets have cardinality 3

$f(n) = n$ and $g(n) = 1$



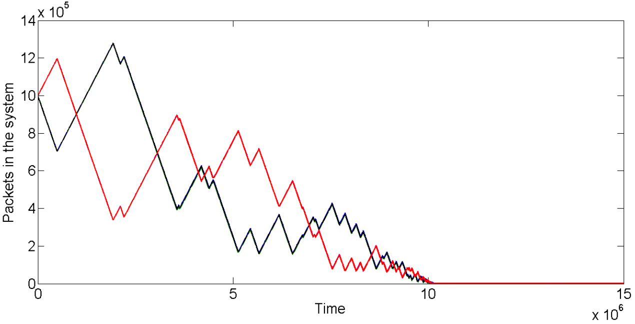
Example of slow mixing

2-partite complete interference graph



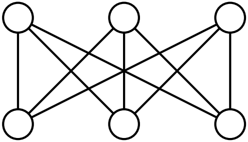
Subsets have cardinality 3

$f(n) = \sqrt{n}$ and $g(n) = 1$



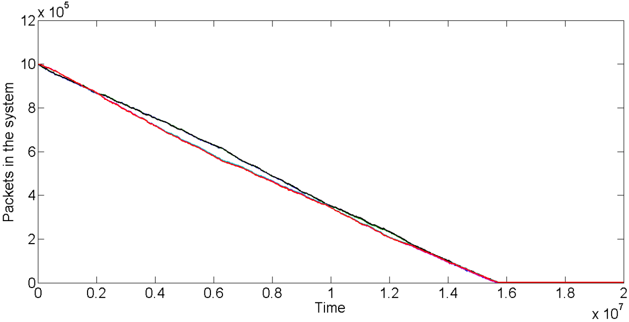
Example of fast mixing

2-partite complete interference graph



Subsets have cardinality 3

$f(n) = \log(n+1)$ and $g(n) = 1$



Conclusion

Backlog-based CSMA algorithms have been shown to guarantee maximum stability, if activity factors behave as logarithmic functions of the backlogs

We showed that more aggressive access schemes can improve the delay performance

How fast are the activity factors allowed to grow, depending on the topology, while retaining maximum stability?

As a first step we investigated fluid limits and identified three qualitative regimes that can arise