International Workshop on MODELING, ANALYSIS, AND CONTROL OF
COMPLEX NETWORKS (San Francisco, USA)

Markov Property of Correlated Random Networks and Its Application to the Analysis of the Internet Topologies

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Background (1)

- Network topology has a large impact on the performance of communication protocols.
- Degree distribution is the most influential.
- However, degree distribution does not fully characterize the network topology.

Background (2)

- Degree correlation
  - Complement to the degree distribution.
  - It really exists in real neworks.

- dK graph (Mahadevan et al.)
  - Network model taking account of degree correlation

P. Mahadevan, et al. ``Systematic topology analysis and generation using degree correlations,'' ACM SIGCOMM, 2006.

dK-distribution and dK-graphs

- dK-graphs: a set of graphs constrained by the dK-distribution

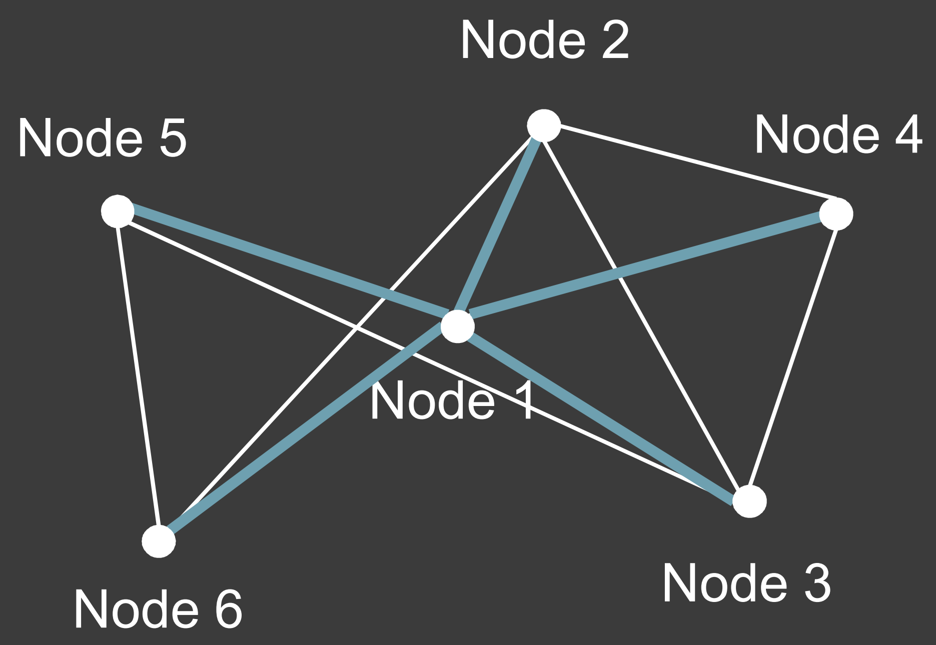
- dK-distribution: joint degree distribution on size-d subgraphs in a given input graph.

dK-Graphs and Constraints

dK graph	Constraints
0K graphs	average degree
1K graphs	degree distribution (or degree sequence)
2K graphs	joint degree-distribution of adjacent nodes (or joint degree sequence)

Example: Degree Sequence

- Input Graph

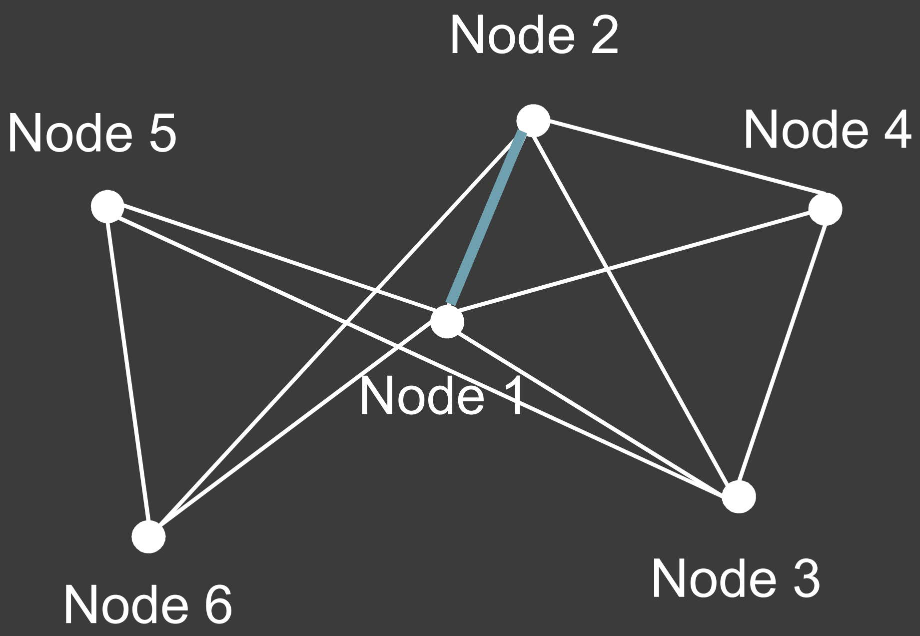


Node	Degree
1	5
2	4
3	4
4	3
5	3
6	3

Degree Sequence

Example: Joint-Degree Sequence

- Input Graph



Link	Joint Degree
Nodes 1 -2	5-4
Nodes 1 -3	5-4
Nodes 1 -4	5-3
Nodes 1 -5	5-3
Nodes 1 -6	5-3
Nodes 2 -3	4-4
Nodes 2 -4	4-3
Nodes 2 -6	4-3
Nodes 3 -4	4-3
Nodes 3 -5	4-3
Nodes 5 -6	3-3

Joint-Degree Sequence

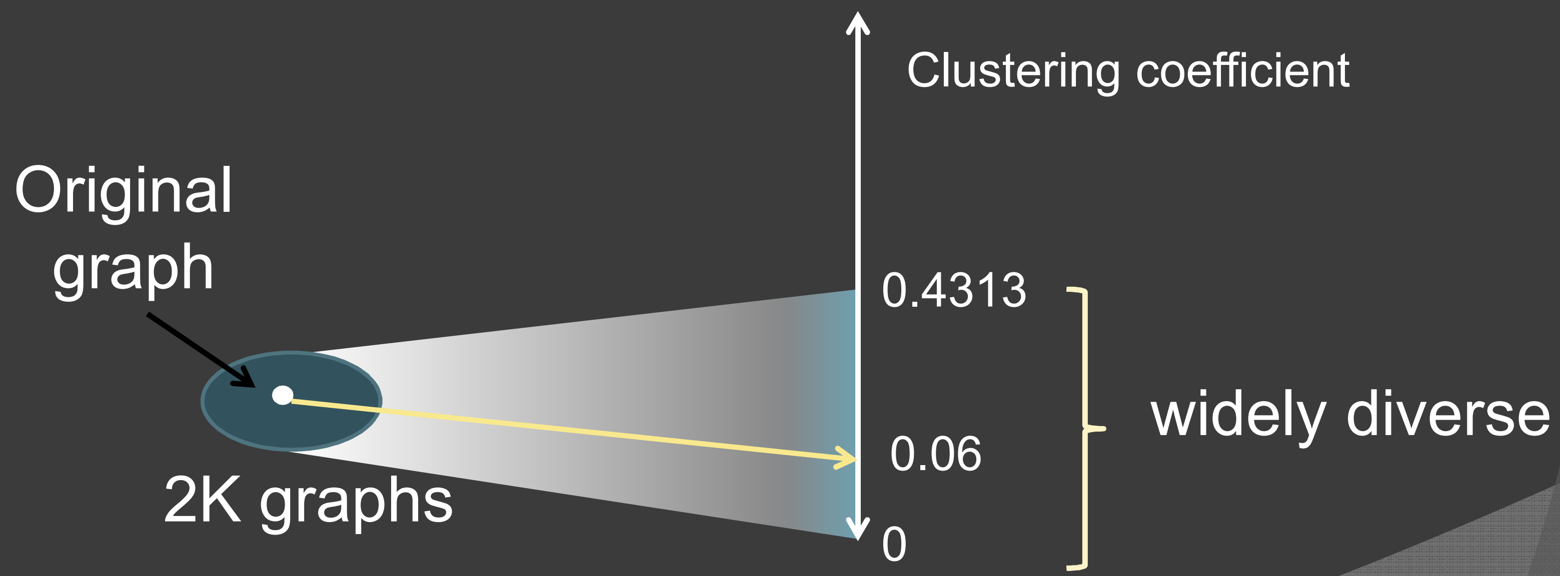
Motivations

- $2K$-graphs would approximate the original graph because joint-degree sequence is a strong constraint.

- However, $2K$ graphs are still widely diverse
  - clustering coefficients have various values.
  - higher order degree correlation

Diversity of $2K$ graphs

- Construct 2K graphs from an original graph with 285 nodes and 226 links.



Maximally Unbiased Graph

- Which graph is typical of 2K graphs?
- Maximally unbiased (mostly random) graph would be typical.

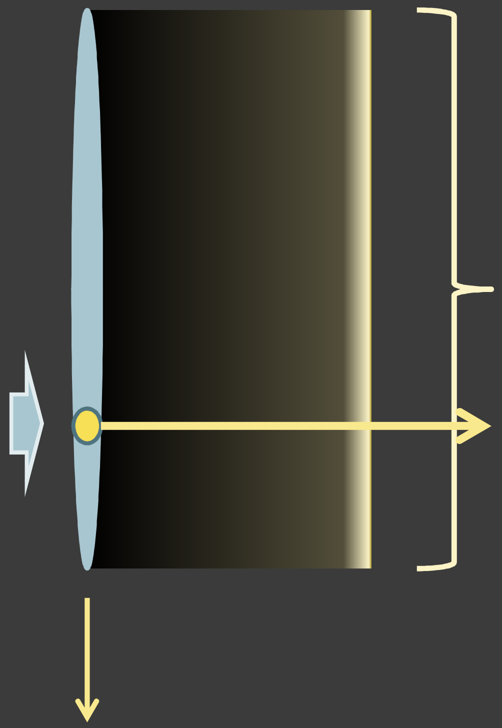
dK graphs	Maximally unbiased graph
0K graph	random graphs
1K graph	uncorrelated networks
2K graph	Markovian networks?

Edge-Based Sampling

- Randomly chooses a link (edge) in the first step and chooses one of its end nodes in the second step.
- $p_e(k)$: degree distribution under the edge-based sampling.
- $p_e(k, l)$: probability that a link has nodes with degrees k and l at its ends.

Joint-Degree Distribution and Uncorrelated Networks

1K graphs



Uncorrelated network

(Joint degree distributions are widely diverse)

$\{p_e(k,l)\}$

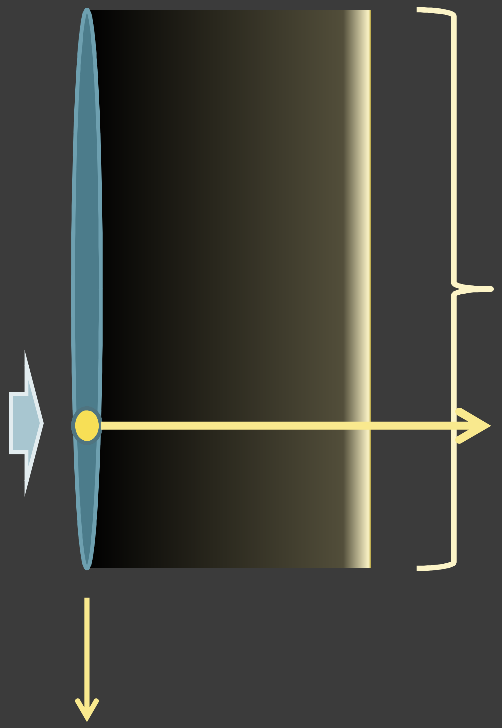
$p_e(k,l) = p_e(k)\, p_e(l)$

(the largest entropy)

$\{p_e(k)\}$ (Constraint: uniquely determined)

Three-Point Degree Correlation and Markovian Networks

2K graphs



Q1: How to model 3-point degree correlation

3-point degree correlations are widely diverse

Markovian network

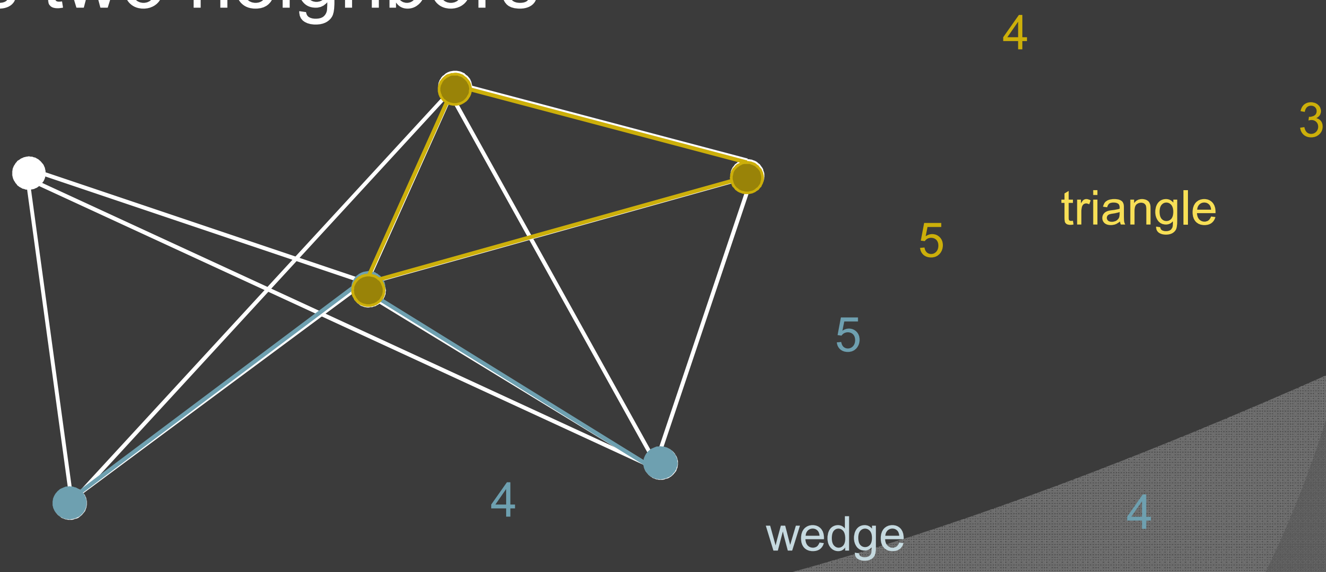
It maximizes the entropy

Q2: Which property should be required for being Markovian

$\{p_e(k,l)\}$ (Constraint: uniquely determined)

Three-Point Degree Correlation (1)

- Degree correlation of a size-three subgraph.
- Size-three subgraph is composed of a node and its two neighbors

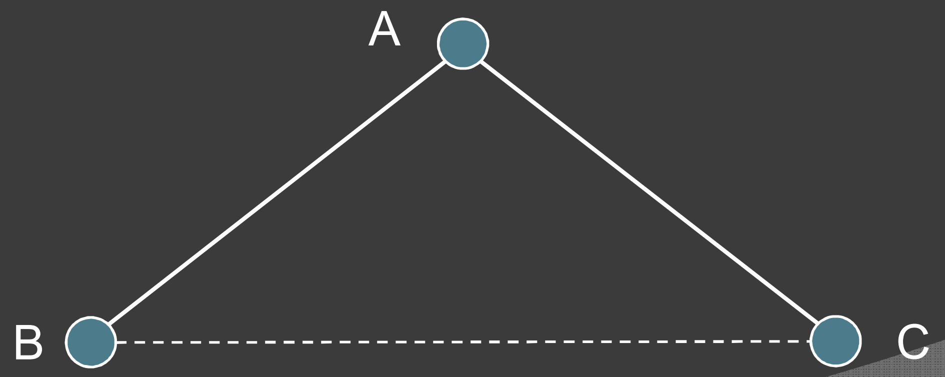


Three-Point Degree Correlation (2)

- Three-point degree correlation is characterized by

$p_e(k, m; l)$: joint degree distribution of nodes A, B, and C

$q(k, m; l)$: probability that nodes B and C are connected.



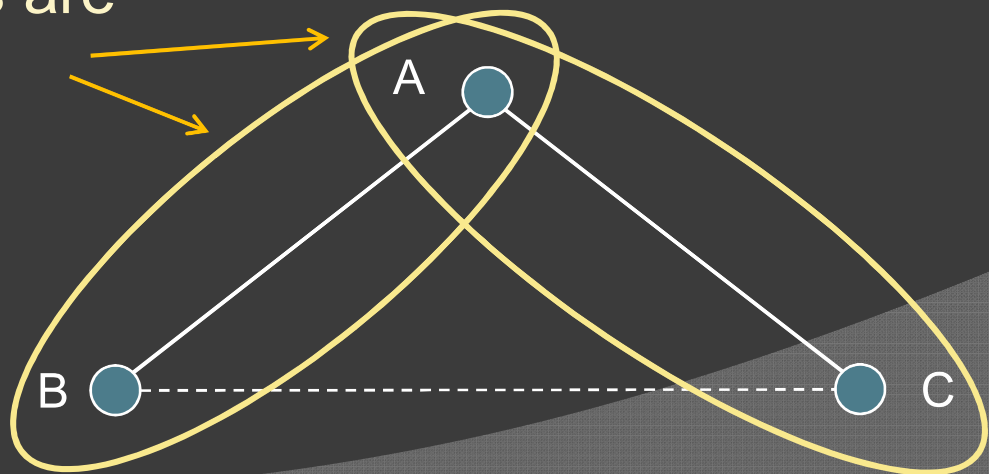
Markov Property (1)

- Condition 1

$$p_e(k,m;l) \propto p_e(k,l)p_e(l,m)$$

※more precisely, $p_e(k,m;l) = \frac{p_e(k,l)p_e(l,m)}{(1-p_e(1))p_e(l)}$

Both correlations are statistically independent



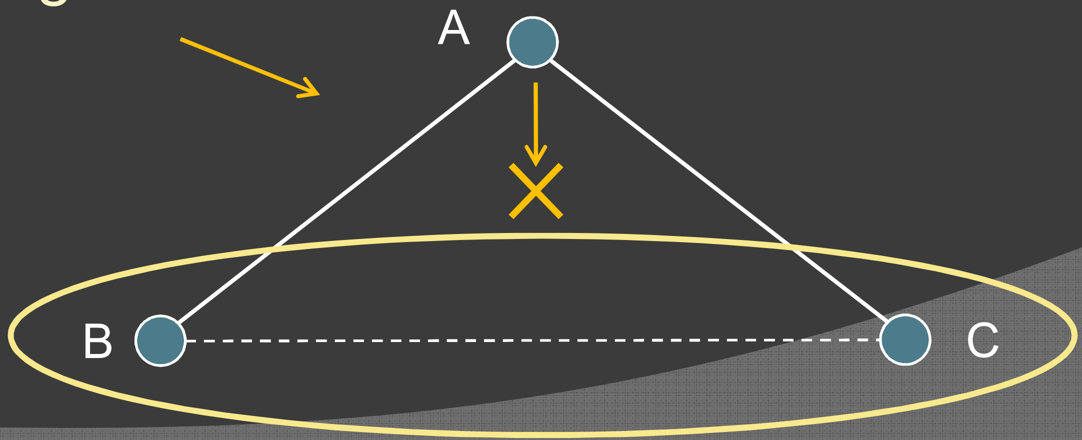
Markov Property (2)

- Condition 2

$$q(k, m; l) = q(k, m)$$

※more precisely, $q(k, m) = \frac{(k-1)(m-1)p_e(k,m)}{NE[D]p_e(k)p_e(m)}$

Whether nodes B and C are neighbors does not depend on the degree of node A



Implications of Markov Property

- Under the assumption of Markov property,

  $p_e(k, m; l)$ and $q(k, m; l)$

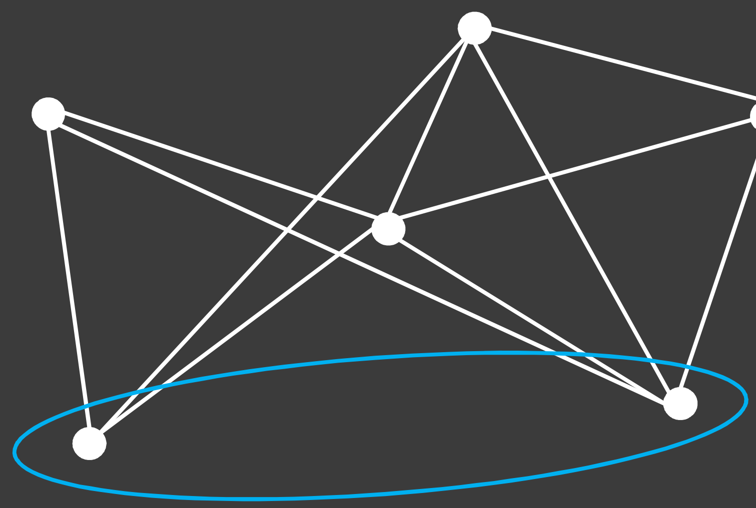
  are fully determined by joint degree distribution $\{p_e(k, l)\}$.

- Topological metrics characterizing three-point degree correlation including

  2nd-order assortativity, clustering coeff.

  are also determined solely by $\{p_e(k, l)\}$.

2^{nd}-order Assortativity



2nd-order assoratativity: Pearson correlation coefficient of the degrees of two nodes located at a two-hop distance.

Internet Topologies

- Are the Internet Topologies Markovian?

- Focus on AS-level and router-level topologies.

  - AS-level topologies: obtained from BGP routing tables collected by RIPE NCC RIS project (available on Web).

  - Router-level topologies: measured by a research group of Washington University (available on Web).

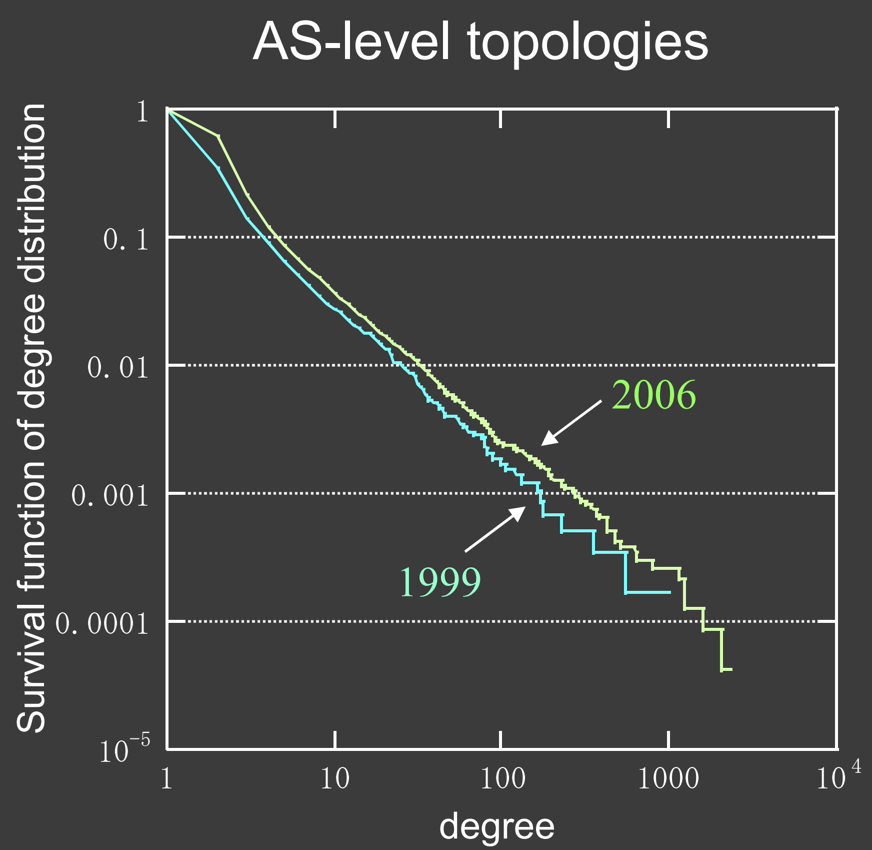
AS-Level Topologies

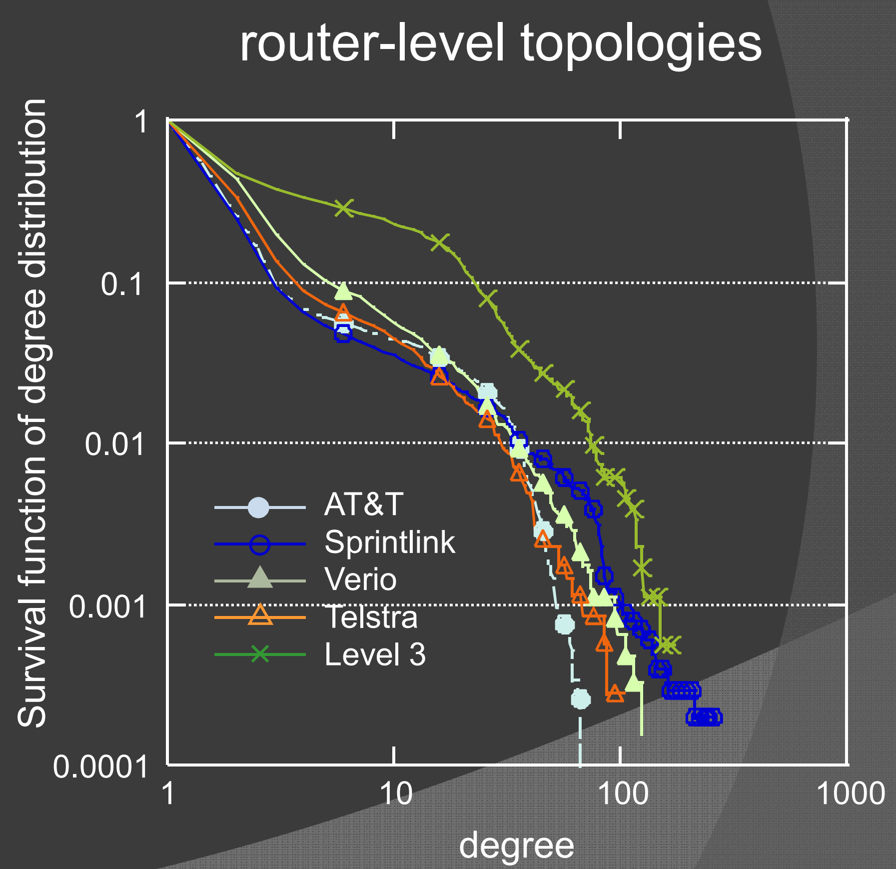
Year	# of links	# of nodes	1st assortativity
1999	7825	5817	-0.1745
2000	16814	8594	-0.1847
2001	22360	11816	-0.1877
2002	25385	13739	-0.1962
2003	29801	15871	-0.1952
2004	34185	18100	-0.1953
2005	37811	20534	-0.1058
2006	43357	23149	-0.1893

Router-Level Topologies

Network	# of links	# of nodes	1st assortativity
AT&T	14261	11745	-0.4501
Sprintlink	12816	10180	-0.3161
Verio	9450	6252	-0.2790
Telstra	4322	3515	-0.2304
Level3	6917	1786	0.0150
Abovenet	1332	654	-0.1964
Tiscail	756	506	0.0627
Exodus	893	424	-0.2109
Ebone	548	300	-0.1985
VSNL	285	226	-0.2359

Degree Distribution

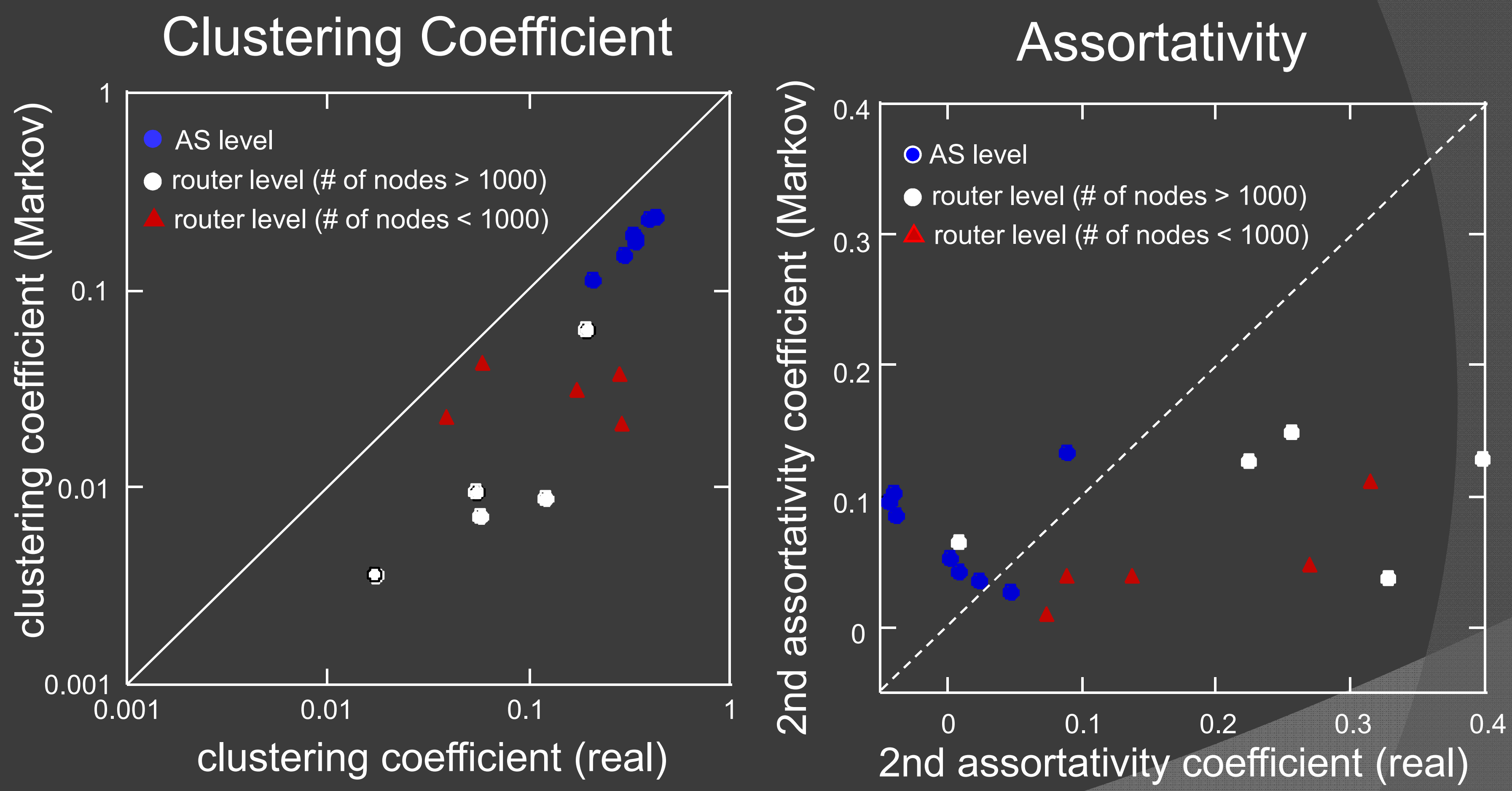




Analysis

- Investigated the joint degree distribution $\{p_e(k,l)\}$ of AS- and router-level topologies.
- Estimated the clustering coefficient and 2nd-order assortativity based on $\{p_e(k,l)\}$ by assuming Markov property.
- Compared the estimates with the actual values.

Results


Clustering Coefficient
AS level
router level (# of nodes > 1000)
router level (# of nodes < 1000)
clustering coefficient (Markov)
clustering coefficient (real)
1
0.1
0.01
0.001
0.001
0.01
0.1
1
Assortativity
AS level
router level (# of nodes > 1000)
router level (# of nodes < 1000)
2nd assortativity coefficient (Markov)
2nd assortativity coefficient (real)
0.4
0.3
0.2
0.1
0
0
0.1
0.2
0.3
0.4

Conclusion

- Markov property is used to know maximally unbiased networks under the constraint of joint-degree distribution.
- Internet topologies studied are not Markovian, meaning the existence of some hidden parameters (e.x. real location) other than degrees.

THANK YOU VERY MUCH!