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A Bi-Criteria Approach for Steiner's Tree Problems in Communication Networks

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1. Introduction

Motivation:

Under **a project financed by Portugal Telecom** for the transport networks optimization:

A **bi-criteria** approach was used to **compute p2p, p2mp and mp2mp virtual connections**with the metrics **hop count** and **load cost**.

In order to:

- balance network traffic;
- minimize the use of network components.

Multiple objective approach:

- Multiple objective formulations enable the trade-offs among different objective functions to be treated mathematically in a fully consistent manner;

- Optimal solution concept is replaced by **non-dominated solution** (which is a solution such that it is not possible to improve one of the objective functions unless at least one of the others is worsened);

- A set of **non-dominated** solutions can be obtained instead of a single solution.

↳ **In which a 'good' compromise solution can be found !**

Previous work:

For **p2p virtual connections** it was shown that:

In general, choosing **compromise solutions** between load cost and hop count allows to balance network traffic and to minimize the use of network components, which **may lead to more carried traffic**.

↓

First proposal **of a bi-criteria heuristic for computing mp2mp virtual connections.**

In this paper we present:

- **An improved version of the a previous proposed heuristic;**

  (suited for application in telecommunication networks whenever it is important to find the minimum amount of resources to connect a given subset of network nodes)

  e. g.: multicast services or mp2mp virtual connections

- **The performance analysis of the heuristics.**

2.A Bi-Criteria Steiner Trees Heuristic

2.1 Review of the first heuristic

Consider:

- The graph G with N vertices and A arcs
- The cost $C_{i,j}$ of the ach arc $(i, j) \in A$

For the terminal nodes $S \subset N$, the problem of finding the shortest tree T_S spanning all nodes in S and possibly some optional nodes in $N \setminus S$ (Steiner nodes) is known as the Steiner's tree problem in graphs

For two additive metrics the tree cost is given by

→ **Bi-criteria Steiner tree(s) problem (BCSTP)**

BCSTP (1-1) heuristic (derived from Kou et al. [5] single criterion heuristic):

1) **Construct the complete undirected graph $G_1(S, A_1)$** such that A_1 is a set of arcs between each pair of nodes in S
(each arc $(i, j) \in A_1$ corresponds to the shortest path $r_{i,j}$ in the graph G);

2) **Compute the second cost of each arc in A_1** by counting the number of times that each arc of A appears in each arc in A_1;

3) **Find the set of efficient supported MST, $\{T_1\}$, of G_1** with the Hamacher et al. algorithm [13];

4) Construct the set of sub-graphs $\{G_S'\}$ of G by **replacing each arc of each T_1 by its corresponding shortest path in G;**

5) Find the set $\{T_S'\}$ of new MST by **removing the cycles in each G_S'**;

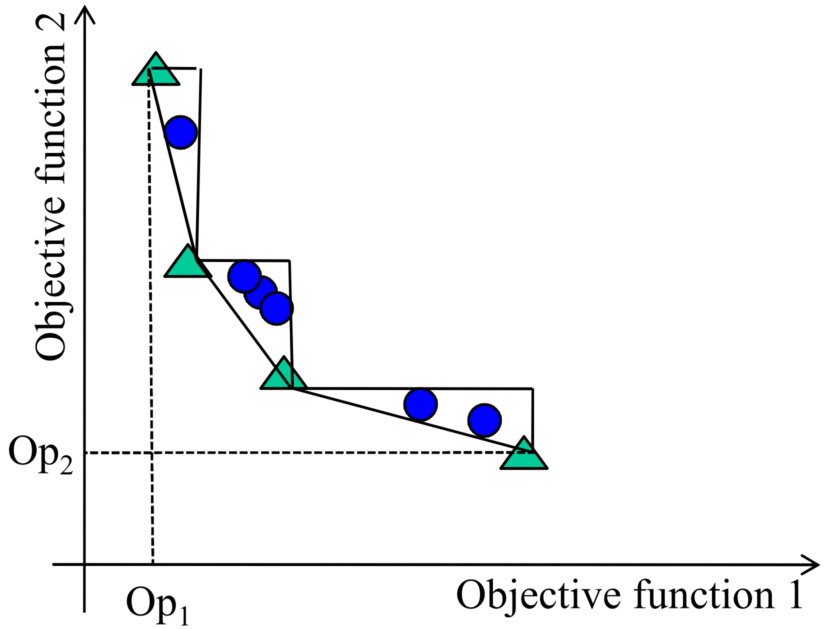
6) Construct the set $\{T_S\}$ from $\{T_S'\}$ by **removing unnecessary arcs** in order that all leaves in the tree are terminal nodes and store it;

7) **Remove all dominated solutions** from the set of all stored $\{T_S\}$ (considering original cost and hop count).

Note that:

- The second metric considered tends to lead to better solutions because, in general, it leads to the inclusion of arcs (in G_2) which have in common a greater number of arcs in G hence promoting the consideration of lower cost Steiner trees also with a lower hop count;

- The Hamacher et al. algorithm [13] determines only **supported non-dominated solutions**and not all non-dominated ones (which is a NP-hard problem).



Supported non-dominated solutions

Unsupported non-dominated solutions

Supported non-dominated solutions can be efficiently computed using the weighted sum method (proposed in Hamacher et al. [13]).

By knowing that some optimal solutions can not be obtained in G1 graph:

- **Parallel arcs in G_1 were considered** by recurring to a k–shortest paths algorithm (MPS [20]);
- G_1 is replaced by $G_2(S, A_2)$ with $|A_2| = k|S|(|S| - 1)/2$;
- k should be a small number such as 2 or 3 because the optimal solution for the single criterion STP is sometimes composed of paths of low cost order in the cases where the shortest path for some node pairs is not the best option;
- As it can exist many paths with the same cost it is also need to limit the maximum number of paths with the same cost to be considered.

BCSTP (2-10)

2.2. A New Bi-Criteria Steiner Trees Heuristic

It is mainly based on a different way of computing the G_2 graph:

- There is a set of G_2 graphs instead of a single one, each one being obtained recurring to a special Steiner node $x \in X$, where X is the set of Steiner nodes;

- There are always only two parallel arcs between each pair of terminal nodes;

- For each G_2 graph the first path between each pair of terminal nodes is always the shortest path in G but the second path is obtained using a node $x \in X$ for all node pairs.

The second path computation for each G_2 graph:

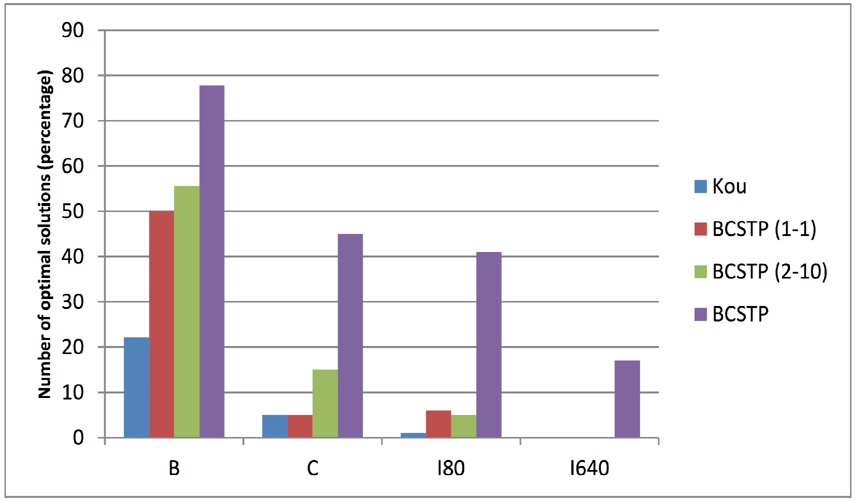
- The tree of the shortest paths between the node x and each terminal node is obtained using Dijkstra algorithm;

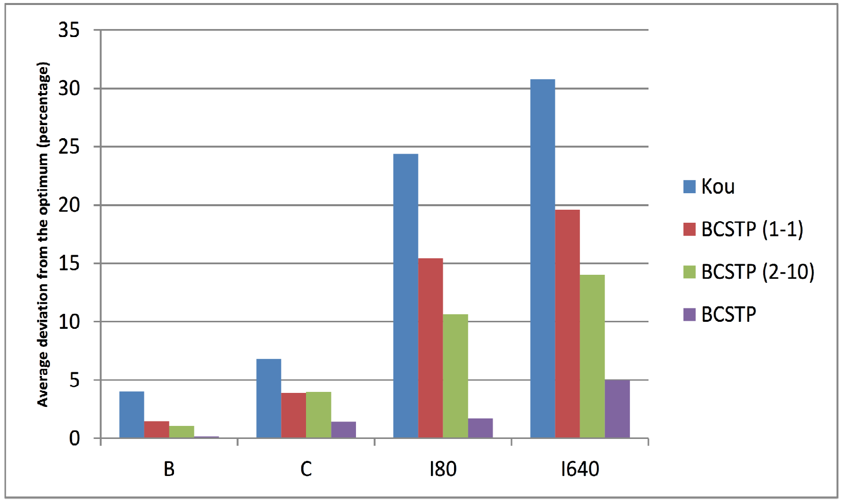
- The second path between each pair of nodes o–d in G is computed as the union of the paths between o and x and between x and d.

3. Results Analysis

The performance evaluation of the heuristics is based on benchmark graphs from the the SteinlibTestdata Library [6].

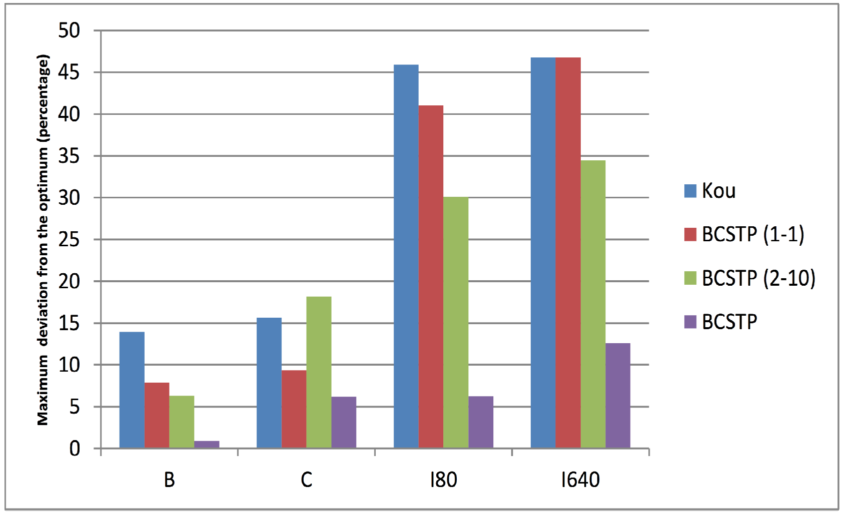
In the following figures the aggregated results for all the networks of the sets B, C, I80 and I640 test graphs were presented for the two versions of BCSTP heuristic.

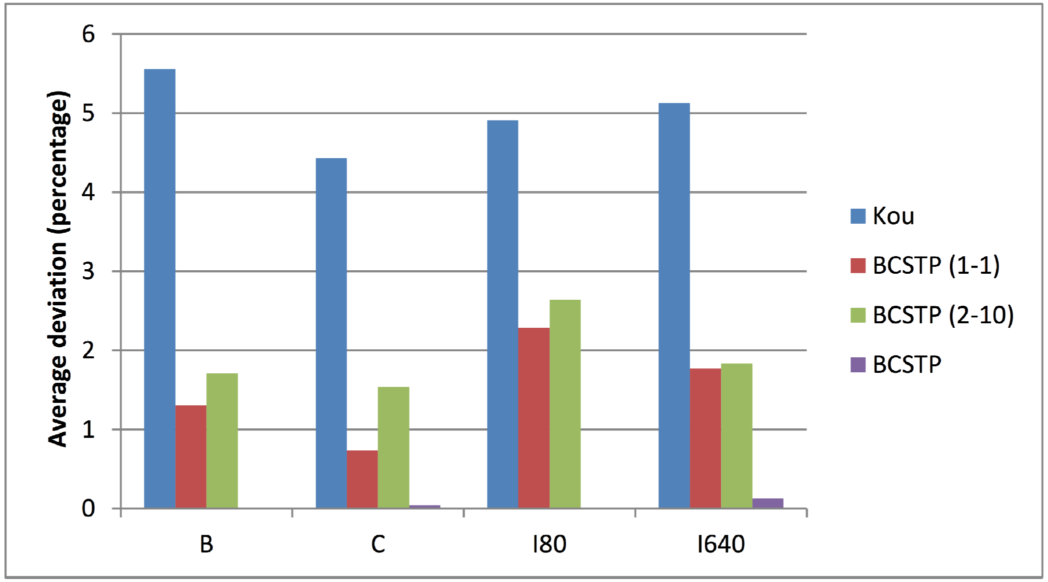




G_2 obtained with the first metric

Best first metric solution
(from the set of all non-dominated solutions)

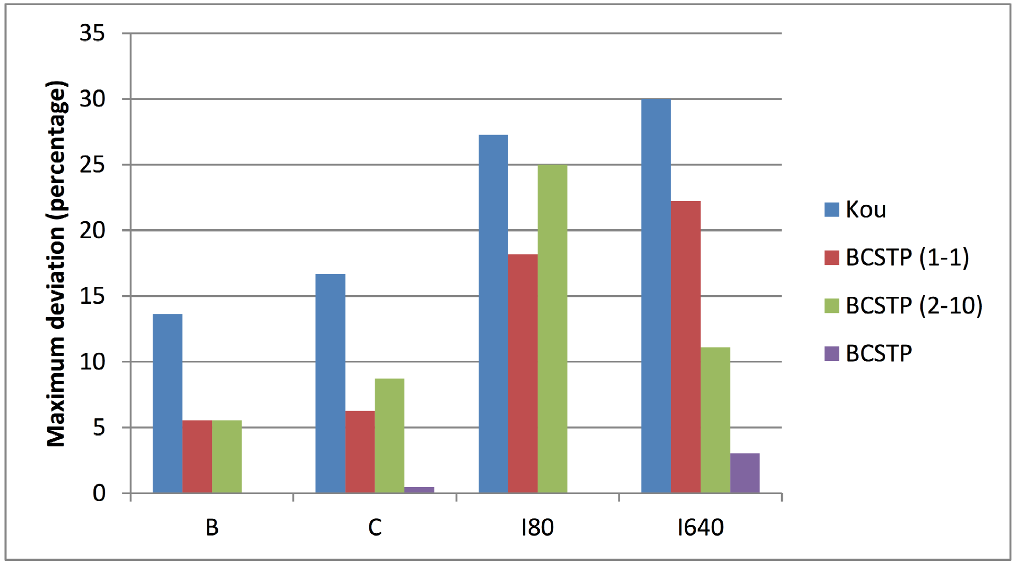




G_2 obtained with the hop count

Best hop count solution
(from the set of all non-dominated solutions)

The optimal solution is unknown



An important aspect in the new BCSTP heuristicis the number of Steiner nodes that is worth to consider (from the set of all Steiner nodes *X*).

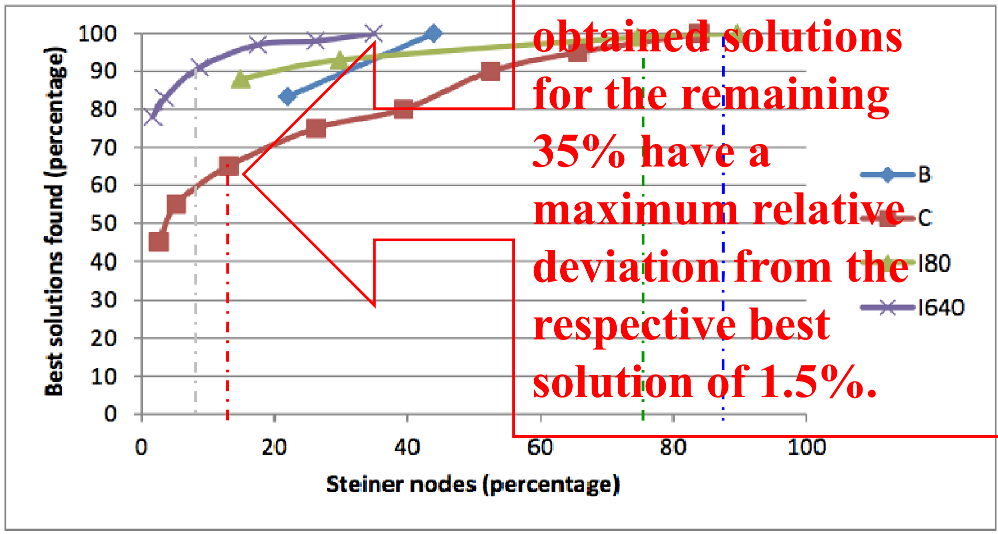
Initially all the Steiner nodes were considered:

- These were the best results that could be obtained;
- With a high computation time.

If the Steiner nodes were ordered according to the cost of its rooted tree (obtained with Dijkstra's algorithm):

- The optimum values were obtained, in general, with low order nodes;
- The best first cost solution improvement obtained with higher order Steiner nodes is, in general, low;

But the number of non-dominated solutions could be different.

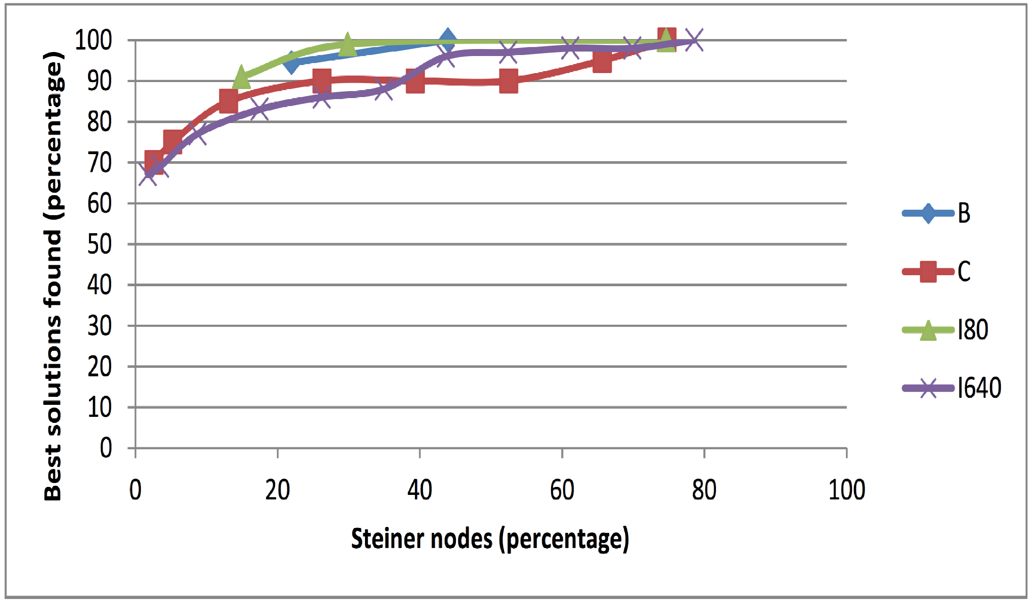


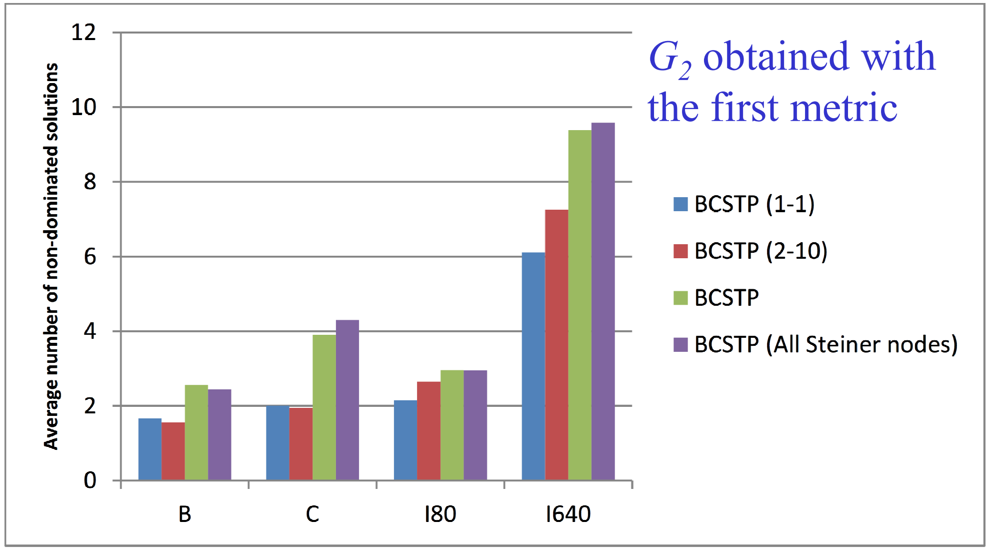
However the obtained solutions for the remaining 35% have a maximum relative deviation from the respective best solution of 1.5%.

- Different numbers of Steiner nodes were considered in the set X (10, 20, 50, 100, 150, 200 and 250);
- Which were presented in the figures as a percentage of the total Steiner nodes for each set of networks.

In the figures are presented the percentage of solutions which were equal to the best solutions obtained with the complete set of Steiner nodes.

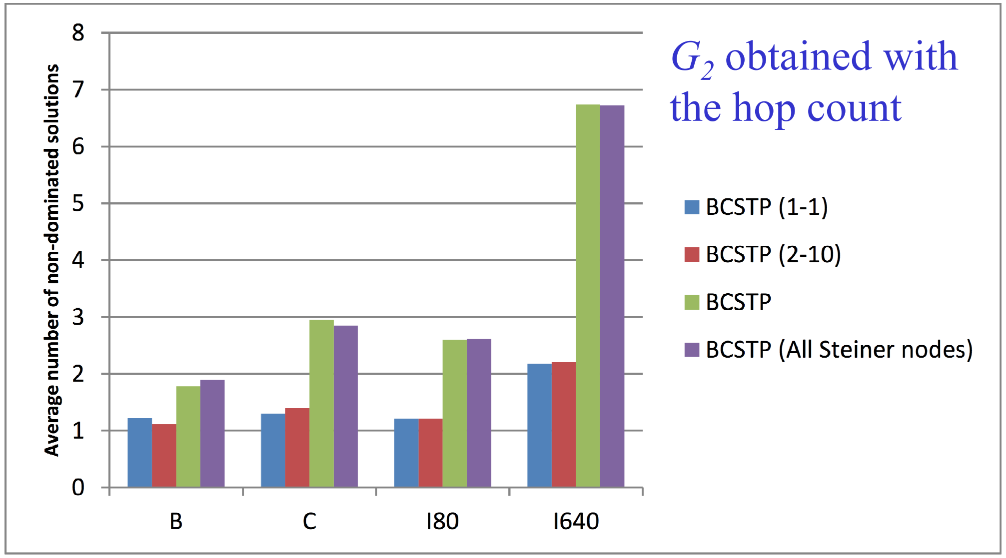
Best hop count solutions





A bi-criteria algorithm give **a set of non-dominated solutions** instead of only one.

The average number of 'non-dominated' solutionsobtained by each algorithm for each set of networks are presented.



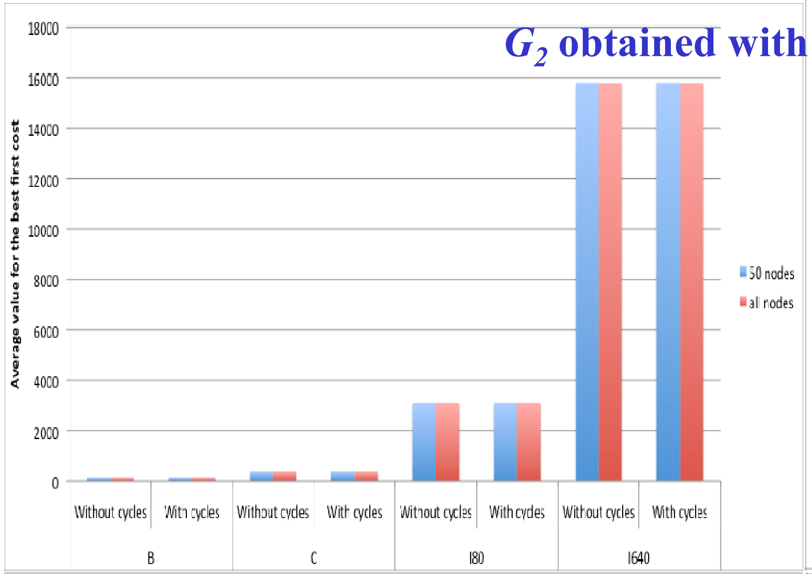
The **elimination of the cycles**in the 5th step of this heuristic is also guided by a bi-criteria decision in order to obtain the maximum possible number of 'non-dominated' Steiner trees in *G*

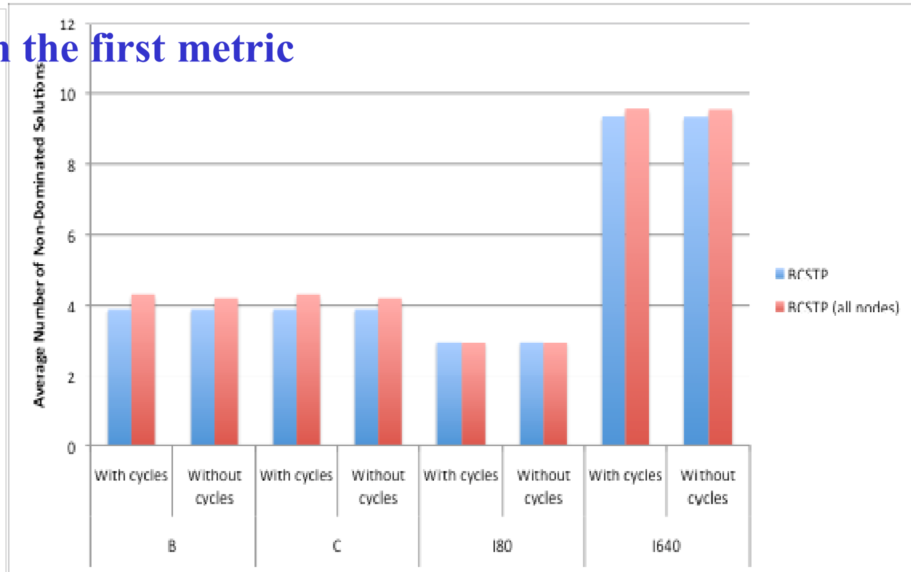
↓

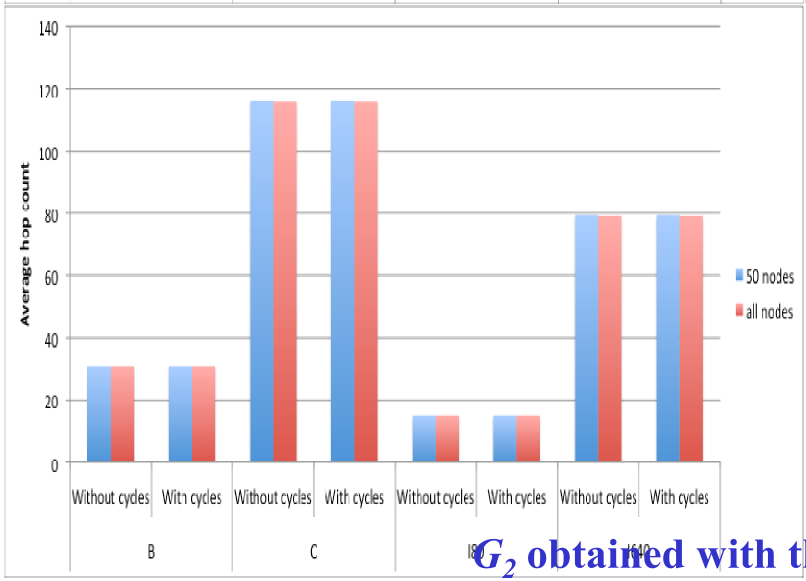
But the results in the next slide show that this is not an efficient strategy!

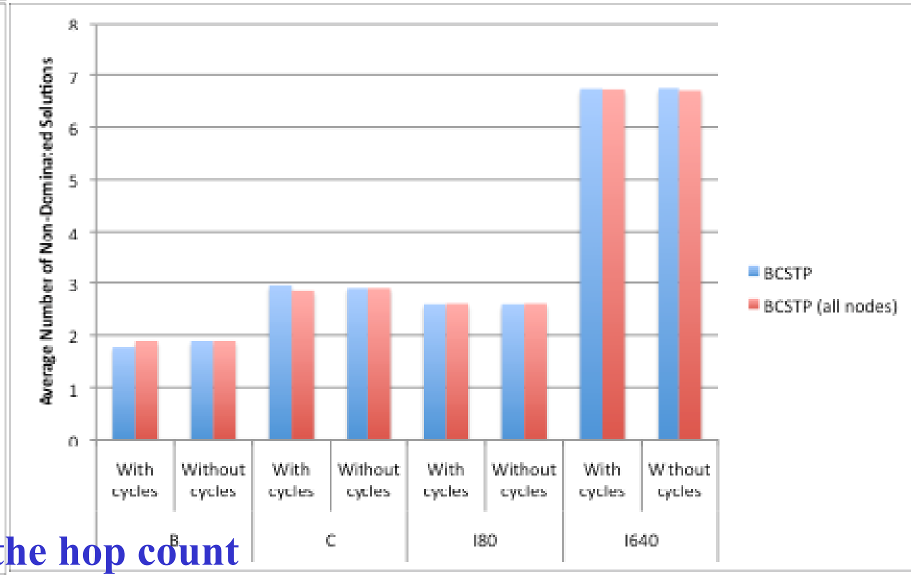
Note that in the next figures 'with cycles' means a 'bi-criteria decision in the elimination of cycles'.

G_2 obtained with the first metric





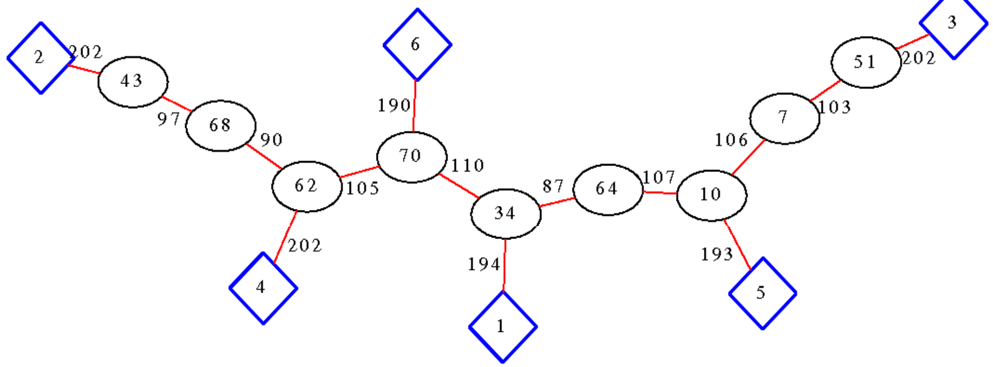


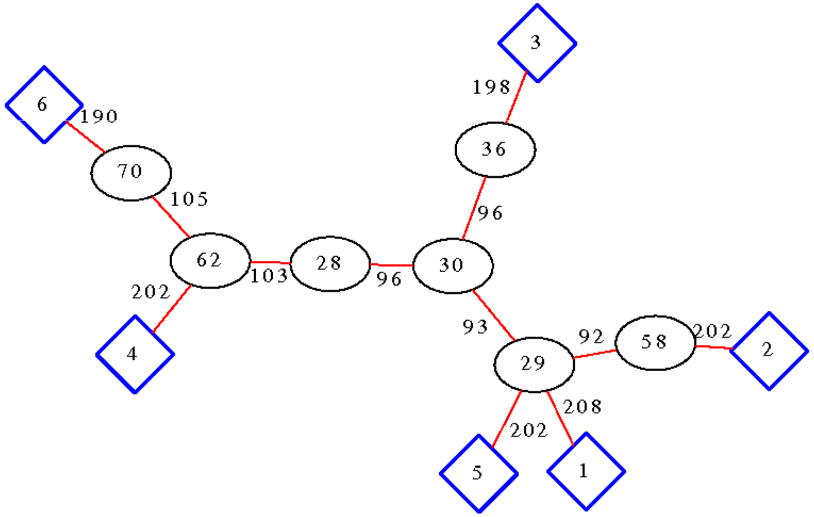


G_2 obtained with the hop count

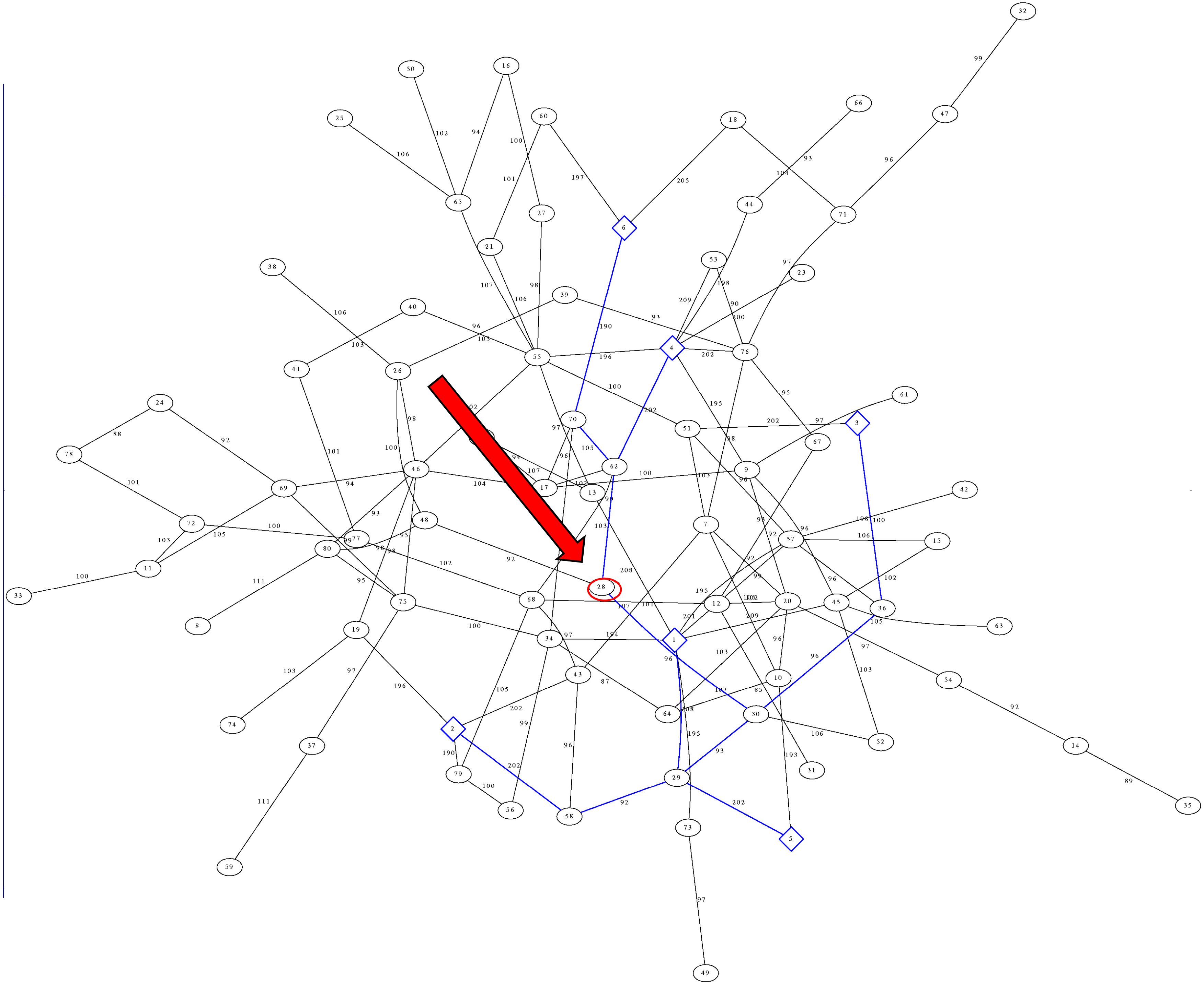
Network i080-001	G2–first metric	G2–hop count
BCSTP (2–10)	(1988,14) (1994,12)	(2003,12) --
BCSTP (1-1)	(1994,12)	(2003,12)
BCSTP	(1787,12)	(1787,12)

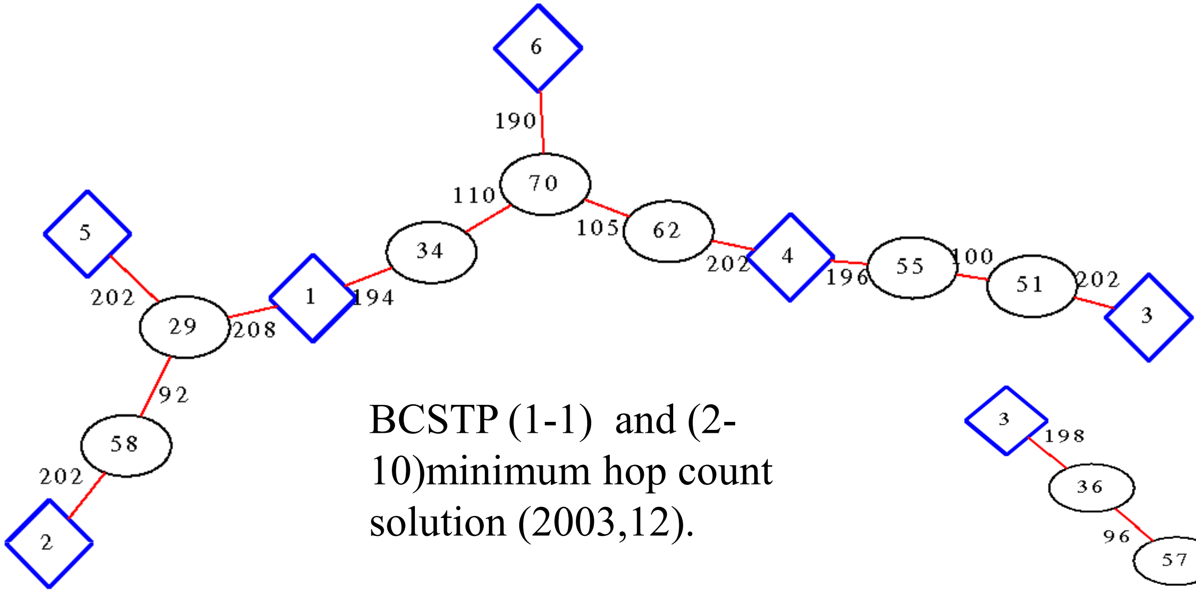
BCSTP (2-10) minimum first metric solution (1988,14).



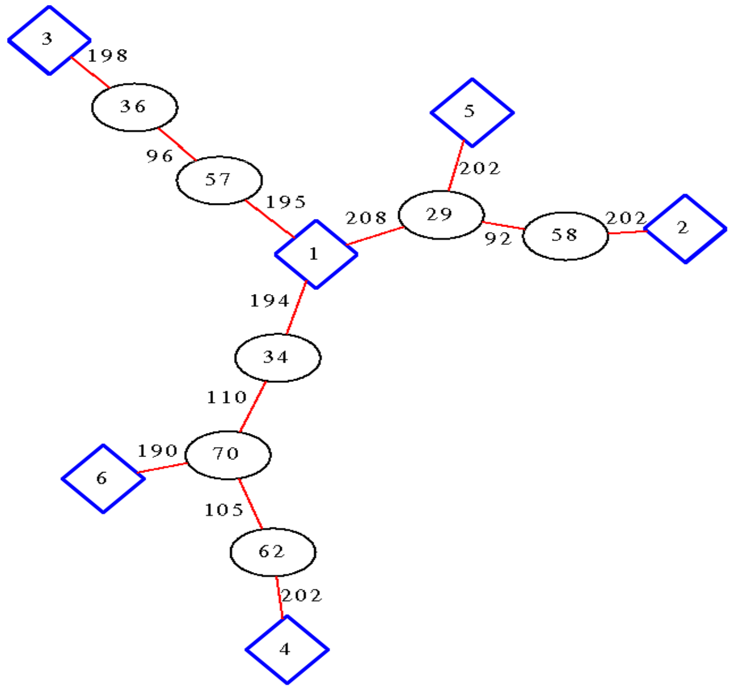


Steinlib optimal solution and BCSTP solution (1787, 12).





BCSTP (1-1) and (2-10)minimum hop count solution (2003,12).



BCSTP(1-1) and (2-10) minimum hop count solution (1994,12).

Average Computation time (in seconds)

	Min.	Average	Max.
BCSTP			
B	0.2	5	25
C	0.5	1874	32865
I80	0.1	0.5	3
I640	0.8	258	2295
BCSTP (2-10)			
B	0.1	8	65
C	0.1	14260	99490
I80	0.01	0.5	8
I640	0.1	2258	22584

4. Conclusions

A new version of a bi-criteria Steiner's trees heuristic was presented

- Well suited for application in communication networks whenever it is important to find the minimum amount of resources to connect a given subset of network nodes;
- Which adds the hop count as a second metric.

The results obtained with reference networks show that:

- It can find the optimal solution for the single criterion STP in some very complex Steiner's tree problems;
- It can also find other solutions with a higher cost but a lower hop count that can be more advantageous in some practical communication network management conditions.